Progress in Celestial Holography

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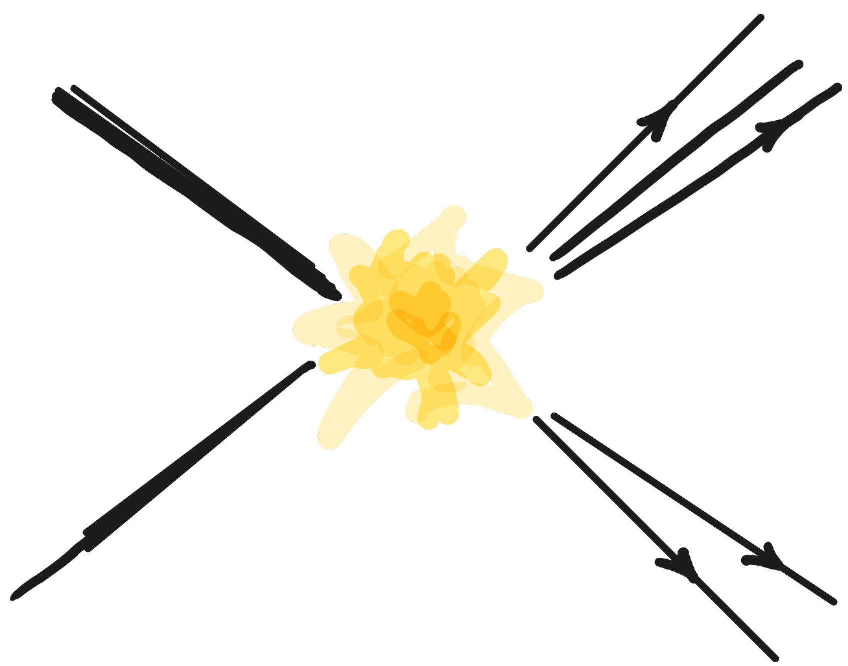
Based on: 2012.04208 with N. Arkani-Hamed, M. Pate and A. Strominger
2104.13432 with A. Atanasov, W. Melton and A. Strominger

Motivation

- What are the observables of quantum gravity?

1. Effective field theory perspective: at scales $E \ll M_{Pl}$



$$\sim \sum_{r \leq m} (G_N s)^m a_{r,m}(s,t) \log^r\left(\frac{s}{M_{Pl}^2}\right) + \ldots$$

- perturbatively calculable;
- predictive, contain information about deflections, scattering angles, etc.

Motivation

- What are the observables of quantum gravity?

2. For $E \sim M_{Pl}$ we don't know...

- At high enough center of mass energies black holes can form and evaporate; perturbation theory breaks down.

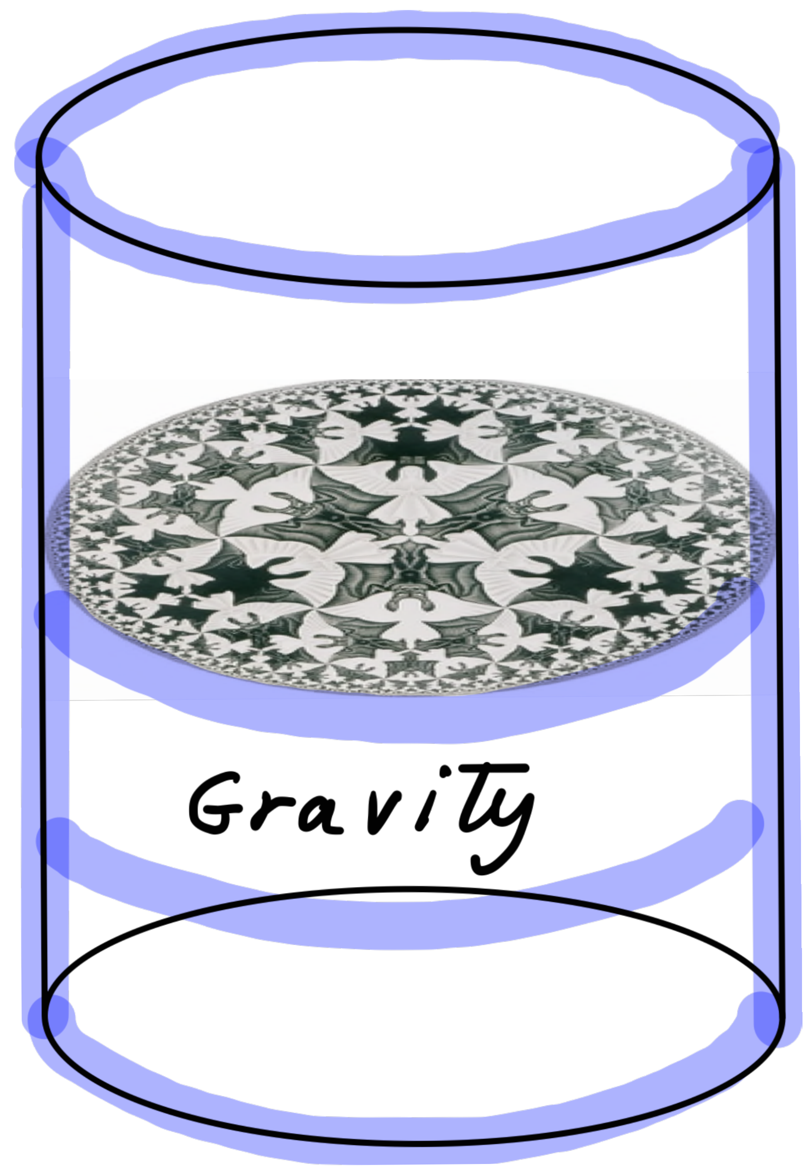
Motivation

- What are the observables of quantum gravity?

2. Answer in AdS quantum gravity provided by holography:

Observables are encoded in correlation functions of a conformal field theory on the boundary.

Many insights: black hole microstates, bulk unitarity/locality/causality, spacetime emergence, ...



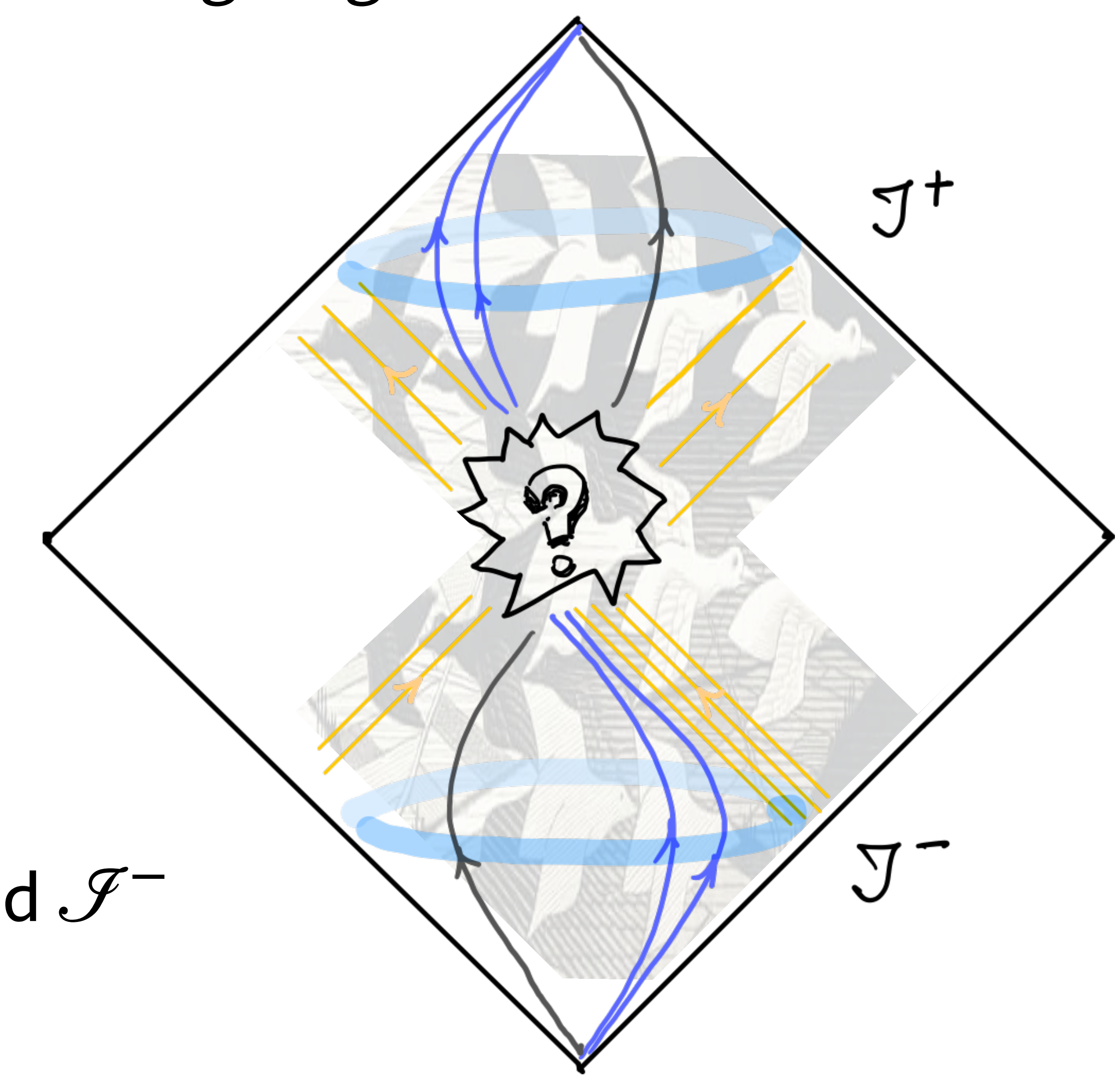
Motivation

- What about asymptotically flat quantum gravity?

  $S_{BH} = \dfrac{A}{4G_N}$ suggests holography should again give the answer, but:

- Boundaries are null - hard to formulate a quantum field theory on $\mathscr{I}$.

- Data at $\mathscr{I}^+$ and $\mathscr{I}^-$ are related by S-matrix; moreover, data on $\mathscr{I}^+$ and $\mathscr{I}^-$ are constrained.



Motivation

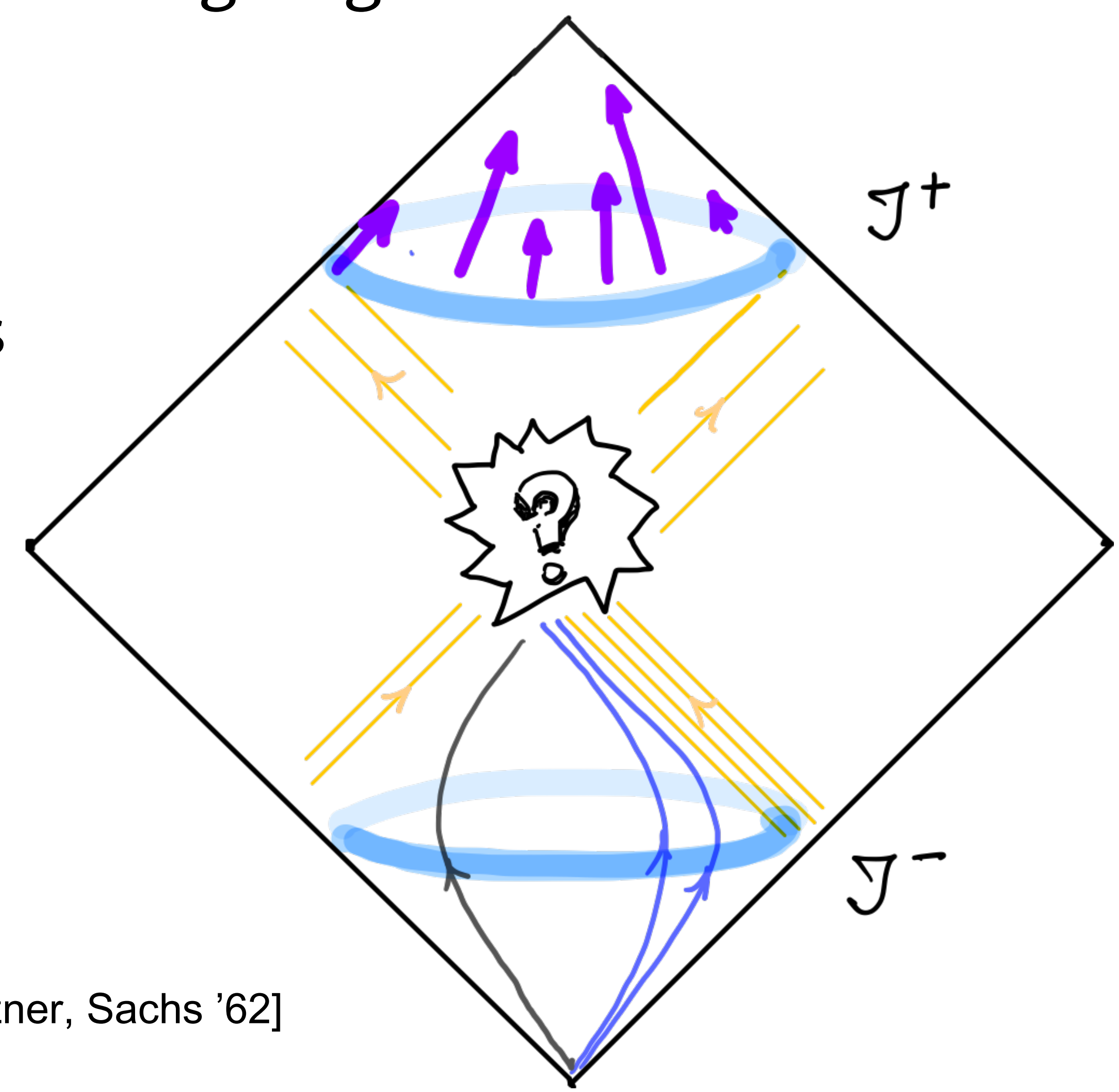
- What about asymptotically flat quantum gravity?

  $S_{BH} = \frac{A}{4G_N}$ suggests holography should again give the answer, but:

- Radiation $\Longrightarrow$ conserved quantities are subtle to define;

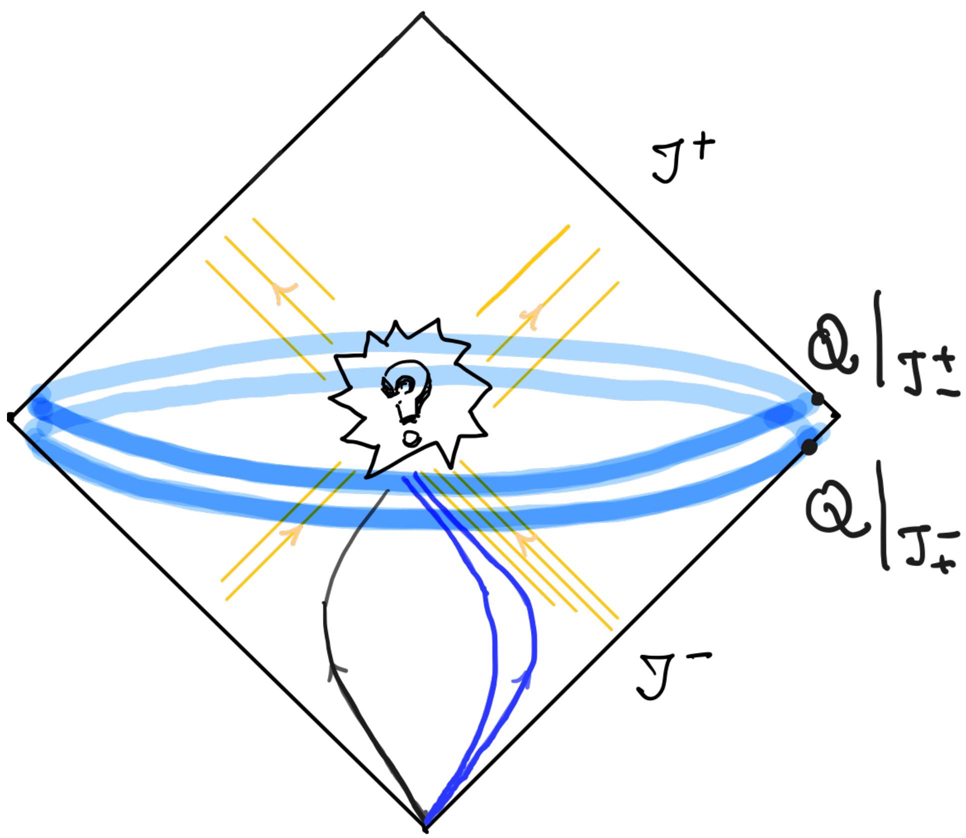
  [Lee, Iyer, Wald, Zoupas '90s]

- Infinite dimensional asymptotic symmetry group $\Longrightarrow$ S-matrix ill-defined, $|\mathrm{out}\rangle = S|\mathrm{in}\rangle$ **up to** supertranslations. [Bondi, van der Burg, Metzner, Sachs '62]



Asymptotic symmetries

- Ambiguity fixed by matching conditions. [Strominger '13]

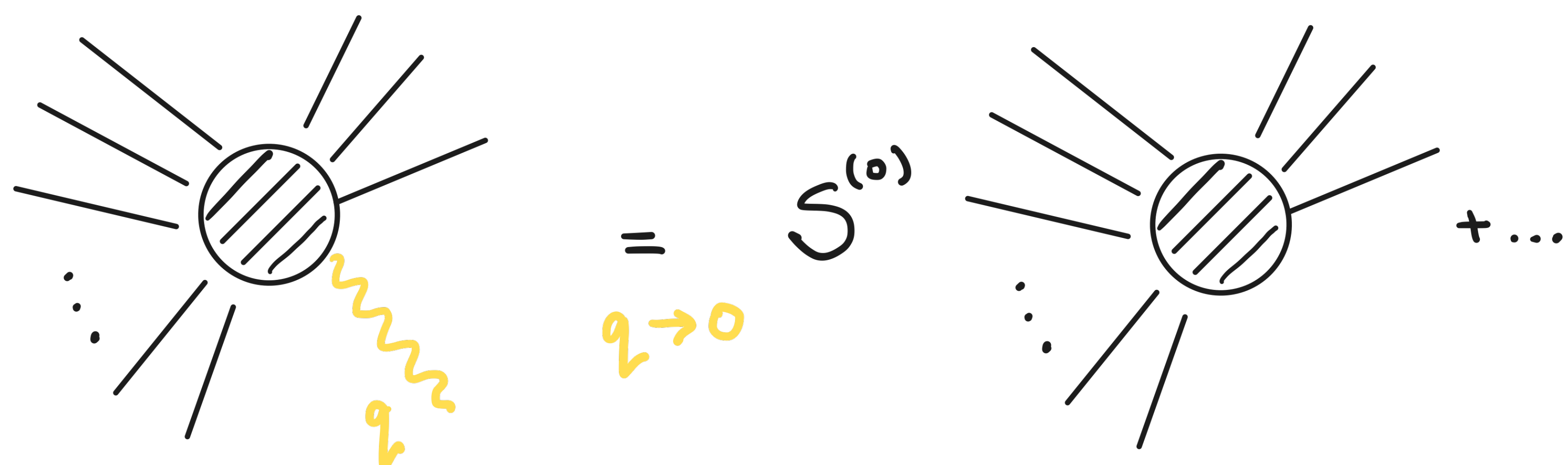


- Eg. supertranslation charges are antipodally matched $Q^+ = Q^-$;

- Their conservation $\langle \mathrm{p_f} | [Q, \mathcal{S}] | \mathrm{p_i} \rangle = 0$ implies the soft graviton theorem!

Soft theorems:

[Weinberg '65]



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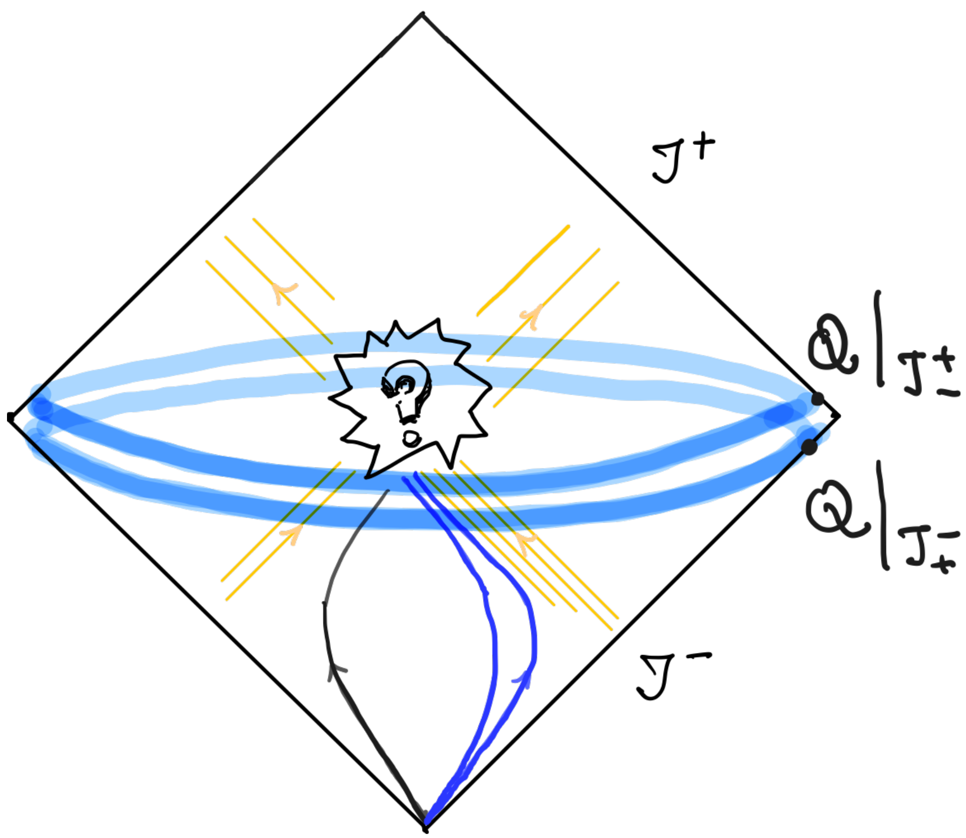

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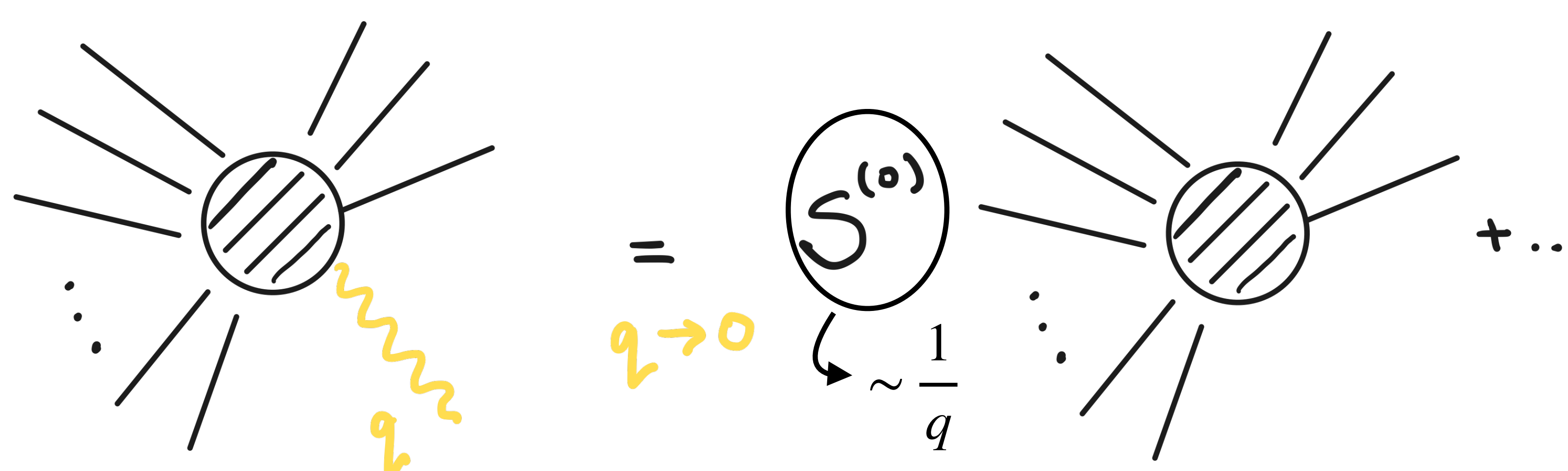

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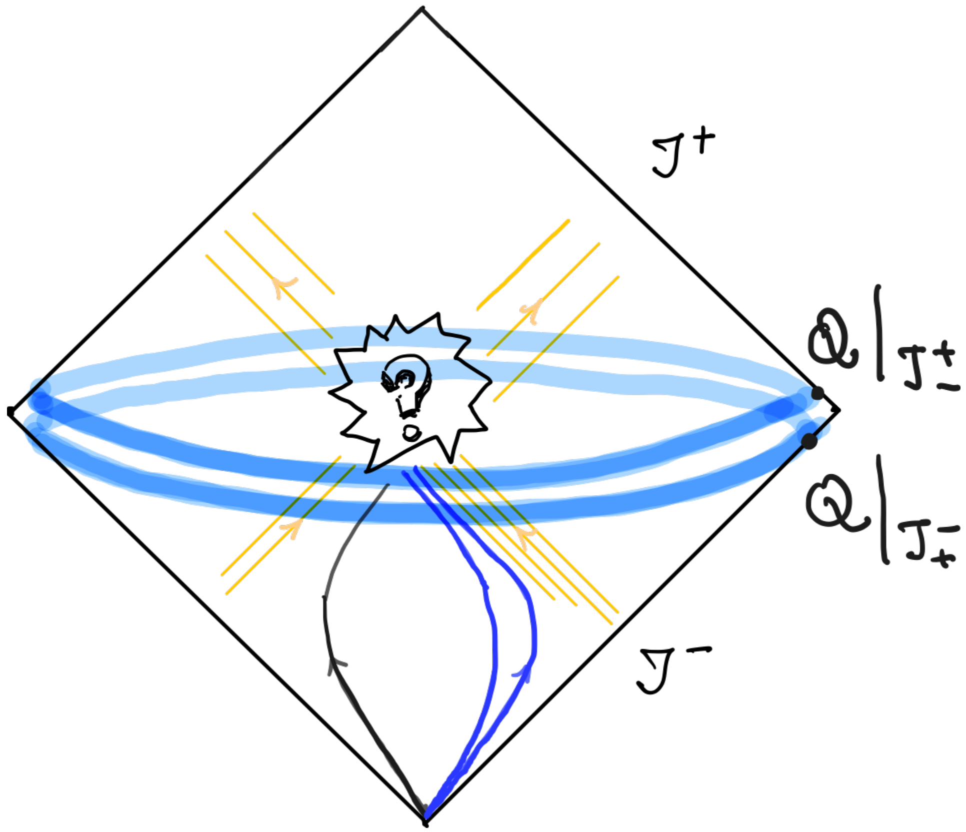


- Subleading soft theorems ⟶ more symmetries? YES!

- Subleading soft graviton theorem ⟶ infinite dimensional enhancement of Lorentz symmetry [Kapec, Lysov, Pasterski, Strominger '16]

- Associated charge conservation implies

$$\langle T_{zz} O_1(\omega_1, z_1, \bar{z}_1) \ldots \rangle = \sum_{i=1}^{n} \left[\frac{\hat{h}_i}{(z - z_i)^2} + \frac{\partial_i}{z - z_i} \right] \langle O_1(\omega_1, z_1, \bar{z}_1) \ldots \rangle,$$

$$\hat{h}_i = \frac{1}{2} \left(s_i - \omega_i \partial \omega_i \right).$$

[Kapec, Mitra, A.R., Strominger '16]

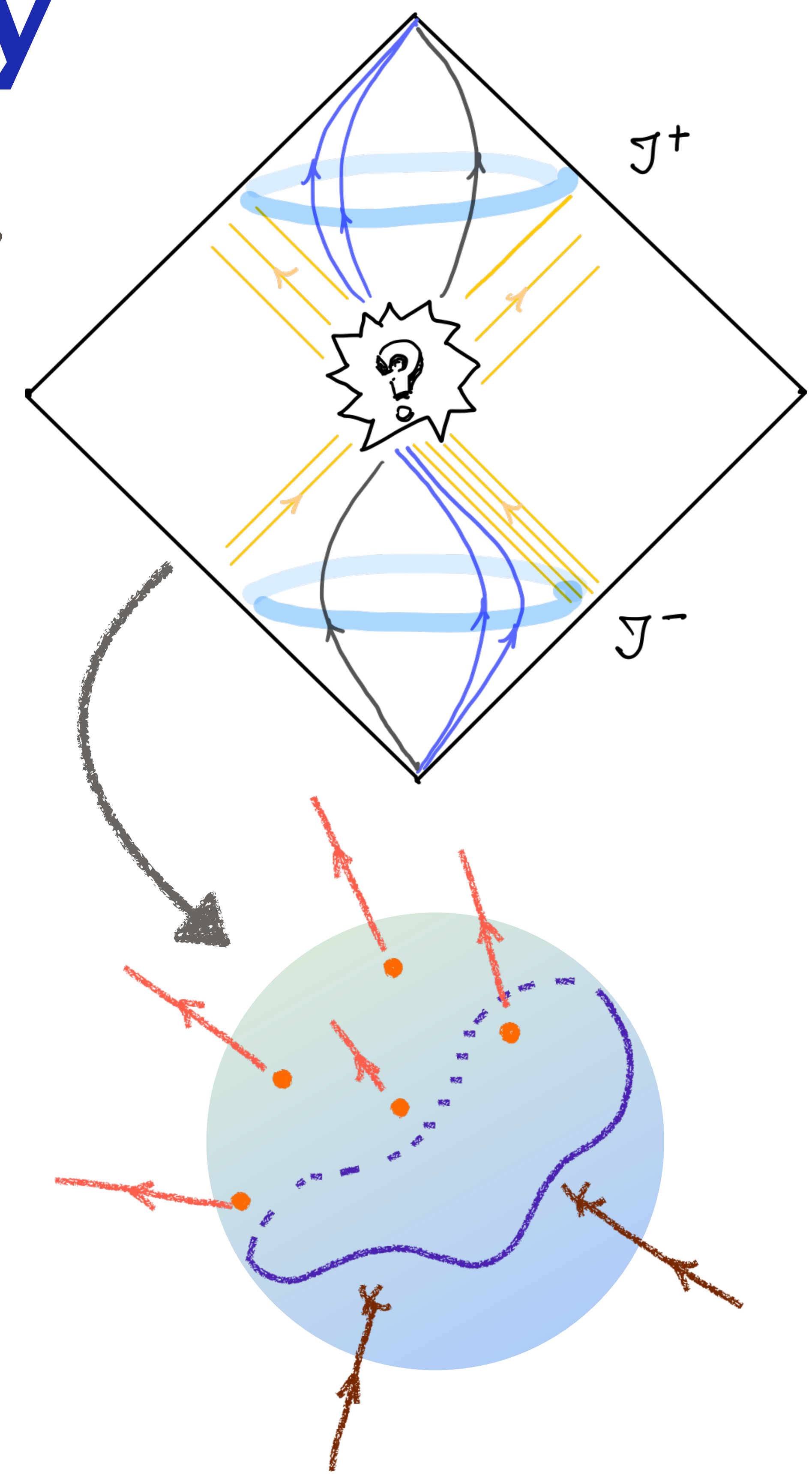
Celestial holography

$$\langle T_{zz} O_1(\omega_1, z_1, \bar{z}_1) \ldots \rangle = \sum_{i=1}^{n} \left[\frac{\hat{h}_i}{(z - z_i)^2} + \frac{\partial_i}{z - z_i} \right] \langle O_1(\omega_1, z_1, \bar{z}_1) \ldots \rangle,$$

$$\hat{h}_i = \frac{1}{2} \left(s_i - \omega_i \partial \omega_i \right) .$$

- Momentum eigenstates don't diagonalize $\hat{h}_i$, but boost eigenstates do!
- Study scattering in basis of boost eigenstates.
- Celestial amplitudes = S-matrices in boost basis
- For asymptotic massless scattering states:

$$| p_i \rangle \rightarrow | \omega_i, z_i, \bar{z}_i \rangle \rightarrow | \Delta_i, z_i, \bar{z}_i \rangle = \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} | \omega_i, z_i, \bar{z}_i \rangle .$$



Novel Features of CCFT

- Lorentz symmetry $\to SL(2,\mathbb{C})$ covariance [Pasterski, Shao, Strominger '17]

- Translations relate celestial amplitudes with shifted weights.

[Donnay, Puhm, Strominger '18; Stieberger, Taylor '18; Law, Zlotnikov '19]

- Soft symmetries associated with conformally soft operators of negative half-integer dimensions and their canonical conjugate Goldstone modes.

[Pate, A.R., Strominger '19; Donnay, Pasterski, Puhm '20; Guevara, Himwich, Pate, Strominger, Strominger '21]

- Soft theorems constrain leading, subleading OPE coefficients in EYM.

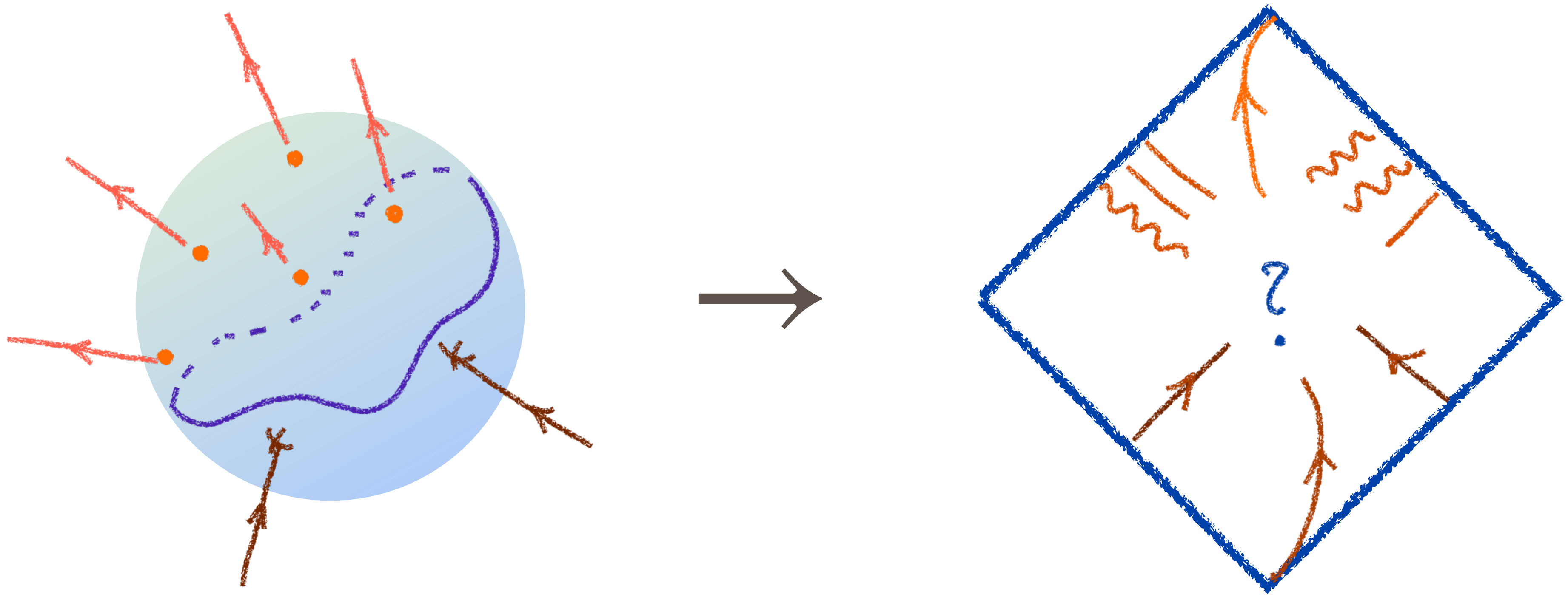
[Pate, A.R., Strominger, Yuan '19; Banerjee, Ghosh '20; Ebert, Sharma, Wang '20]

- IR divergences are captured by vertex operators of Goldstones.

[Nande, Pate, Strominger '17; Himwich, Narayanan, Paul, Pate, Strominger '20]

Outline

Can we learn anything about quantum gravity?

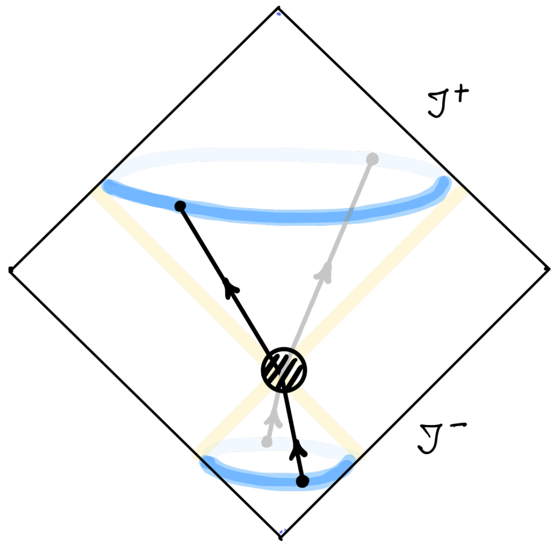


1. Analytic structure of celestial 4-point scattering in the complex boost weight β-plane captures perturbative and non-perturbative aspects of bulk scattering. [Arkani-Hamed, Pate, A.R., Strominger '20]

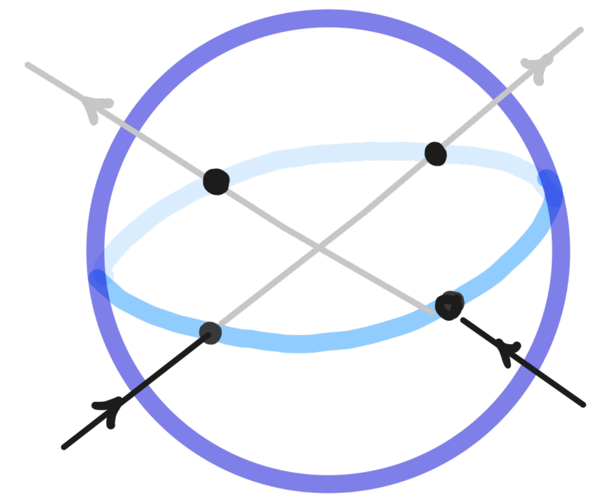
2. Conformal block expansion provides evidence that residues of poles in β can be reconstructed from celestial OPE data.

[Atanasov, Melton, A.R., Strominger '21]

Massless scalar 4-point scattering



Momentum space



Celestial sphere

$$\mathbb{A}(p_i) = \mathscr{M}(s,t)\delta^{(4)}\left(\sum_{i=1}^{4} p_i\right)$$

$$s = -(p_1+p_2)^2 \equiv \omega^2$$

$$t = -(p_1+p_3)^2 \equiv -z\omega^2$$

$\rightarrow$

$$\widetilde{\mathscr{A}}(z_i,\bar{z}_i;\beta) = K(z_i,\bar{z}_i)X(z,\beta)\int_0^\infty d\omega\,\omega^{\beta-1}\mathscr{M}(\omega^2,-z\omega^2)$$

$$\beta = \sum_{i=1}^{4}\Delta_i - 4, \qquad z = -\frac{t}{s} = \frac{z_{13}z_{24}}{z_{12}z_{34}} \in [0,1]$$

$$K(z_i,\bar{z}_i) = \prod_{i<j} z_{ij}^{\frac{h}{3}-h_i-h_j}\bar{z}_{ij}^{\frac{\bar{h}}{3}-\bar{h}_i-\bar{h}_j}$$

$$X(z,\beta) = \delta(z-\bar{z})\,|\,z(1-z)\,|^{\frac{1}{6}(\beta+4)}$$

Part I:
Analytic structure in β

Imprints of UV physics

- General formula:

$$\widetilde{\mathscr{A}}(z_i, \bar{z}_i; \beta) = \underbrace{K(z_i, \bar{z}_i)X(z, \beta)}_{\text{kinematics}} \underbrace{\int_0^\infty d\omega \omega^{\beta-1} \mathscr{M}(\omega^2, -z\omega^2)}_{\equiv \mathscr{A}(\beta, z),\ \text{dynamics}} .$$

- Consider the (sick) example:

$$\mathscr{M} \propto \omega^p \implies \mathscr{A} \propto \int_0^\infty d\omega \omega^{\beta+p-1} \propto \delta(\beta + p),\ \beta + p \in i\mathbb{R}$$

Scattering amplitudes with poor UV behavior

$\implies$ badly behaved, non-analytic celestial amplitudes.

Analytic structure in β

- General formula:

$$\widetilde{\mathscr{A}}(z_i, \bar{z}_i; \beta) = \underbrace{K(z_i, \bar{z}_i)X(z, \beta)}_{\text{kinematics}} \underbrace{\int_0^\infty d\omega \omega^{\beta-1} \mathscr{M}(\omega^2, -z\omega^2)}_{\equiv \mathscr{A}(\beta, z),\ \text{dynamics}}\,.$$

- $\mathscr{A}(\beta, z)$ is well defined provided $\mathscr{M}$ falls off fast enough as $\omega \to \infty$,

  eg. s-channel pole $\quad \mathscr{M} = \lambda \dfrac{M^2}{\omega^2 - M^2} \implies \mathscr{A} \propto \dfrac{\lambda M^\beta}{\sin \pi\beta/2}.$

- Poles in $\omega \implies$ infinite sequence of poles in β-plane!

Analytic structure in β

- Low-energy expansion with massive states integrated out and no massless loops

$$\mathcal{M}(s,t) = \sum_{\Delta,q} a_{\Delta,q} s^{\Delta-q} t^q \iff \mathcal{M}(\omega^2, -z\omega^2) = \sum_{\Delta,q} a_{\Delta,q} \omega^{2\Delta} (-z)^q \,.$$

leads to celestial amplitude (with cutoff ω_*)

$$\mathcal{A}(\beta, z) = \sum_{\Delta} \frac{\tilde{a}^{IR}_{\Delta}(z)}{\beta + 2\Delta} + \ldots$$

- First pole in β determined by lowest dimension operators in EFT.

- Sequence of poles at increasingly negative even integer β carries information about higher dimension operators; residues obey positivity constraints.

[Arkani-Hamed, Huang, Huang '20]

Analytic structure in β

- If the scattering amplitude also admits an expansion around

$$\omega \to \infty \implies \mathscr{A}(\beta, z) = \sum_{n \in \mathbb{Z}_-} \frac{\tilde{a}_n^{UV}(z)}{\beta + 2n} + \dots$$

- Accounting for loop effects

$$\mathscr{M}(\omega^2, -z\omega^2) = \sum_{m,n;r \le m} a_{m,n,r}(z)(G_N\omega^2)^m \omega^{2n} \log^r \frac{\omega}{\Lambda_{UV}}$$

yields higher order poles in β

$$\mathscr{A}(\beta, z) \supset \int_0^{\omega_*} d\omega \omega^{\beta-1} \log^r \omega \propto \frac{\partial^r}{\partial \beta^r} \frac{1}{\beta} \propto \frac{1}{\beta^{r+1}} .$$

Another UV constraint

- Exponential fall-off in the hard scattering limit

$$\lim_{\omega \to \infty} \mathscr{M}(\omega^2, -z\omega^2) \propto e^{-\omega^2/M^2}\,.$$

- Large β limit localizes the integrand at high energy and

$$\lim_{\beta \to \infty} \mathscr{A}(\beta, z) \to \int_0^\infty d\omega \omega^{\beta-1} e^{-\omega^2/M^2} = \frac{M^\beta}{2}\Gamma(\beta/2)\,.$$

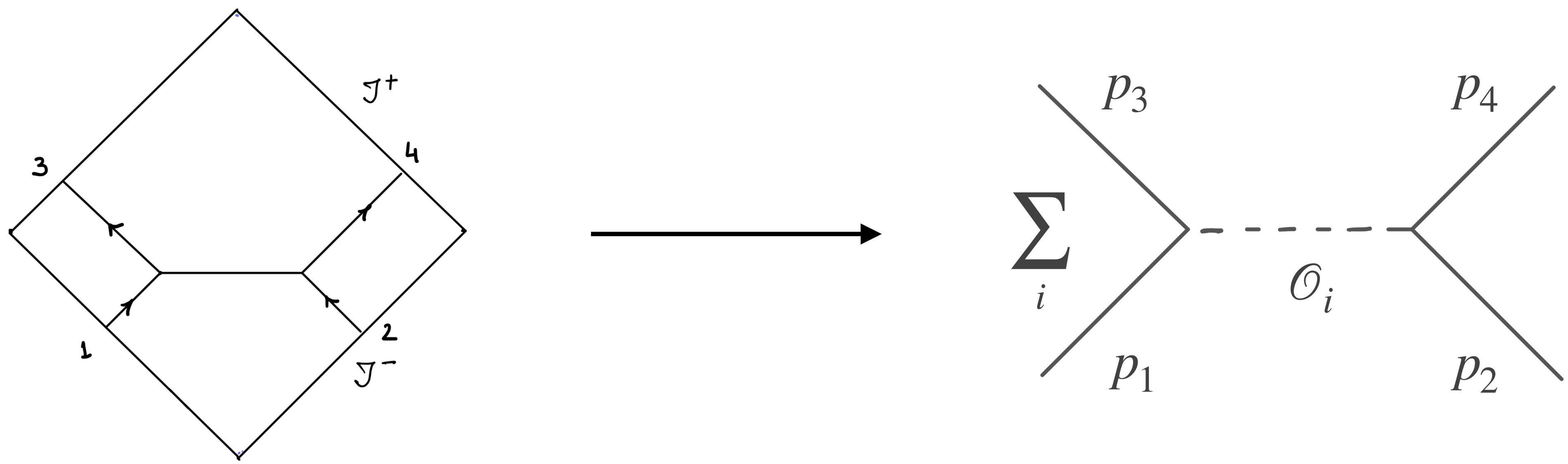
- Examples: black holes $S_{BH} = 4\pi G_N \omega^2,\ M \propto M_{Pl};$

  tree-level string amplitudes $M \propto M_s\,.$

- Absence of poles in the right complex β plane is a sharp signature of quantum gravity!

Part II:
Analytic structure in z

Analytic structure in z



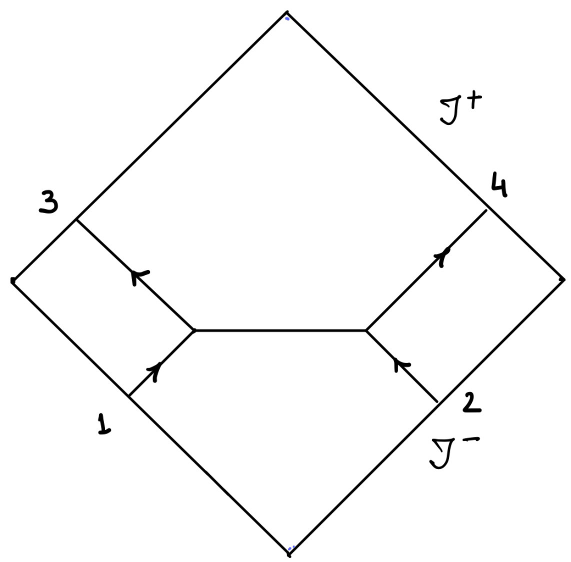
- Tree-level, t-channel scattering $\mathcal{M}(s,t) = -g^2 \frac{1}{t-m^2} \quad \to$

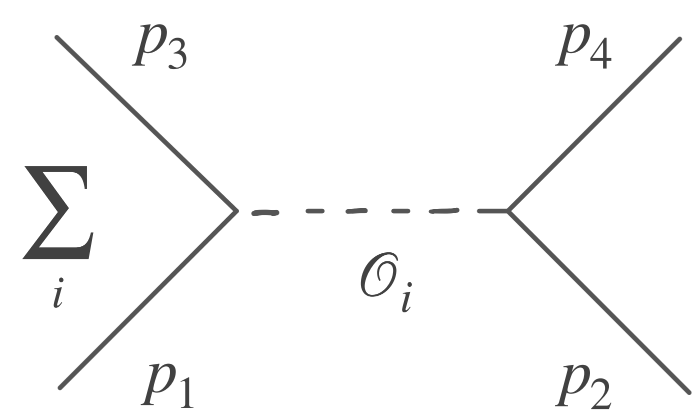
$$\widetilde{\mathcal{A}}(z_i, \bar{z}_i; \beta) = g^2 K(z_i, \bar{z}_i) X(z, \beta) \int_0^\infty d\omega \omega^{\beta-1} \frac{1}{z\omega^2 + m^2}$$

$$= I_{13-24}(z_i, \bar{z}_i) \underbrace{N_{gm}(\beta) |z|^2 |1-z|^{h_{13}-h_{24}} \delta(z-\bar{z})}_{f_t(z,\bar{z})}$$

Goal: Decompose $f_t(z, \bar{z})$ into conformal blocks.

Analytic structure in z





- Tree-level, t-channel scattering $\mathscr{M}(s,t) = -g^2 \frac{1}{t-m^2} \quad \longrightarrow$

$$\widetilde{\mathscr{A}}(z_i, \bar{z}_i; \beta) = I_{13-24}(z_i, \bar{z}_i) \underbrace{N_{gm}(\beta)\,|z|^2\,|1-z|^{h_{13}-h_{24}}\delta(z-\bar{z})}_{f_t(z,\bar{z})}$$

Roadmap: Treat $z, \bar{z}$ as real independent variables and expand $f_t(z, \bar{z})$ on a complete set of orthogonal solutions to the two particle conformal Casimir over the Lorentzian square, $z, \bar{z} \in [0,1]$.

Conformal partial waves

- 1-3 conformal Casimir equation $$\left(\mathscr{D}_z + \mathscr{D}_{\bar z}\right)\Psi_{h,\bar h}(z,\bar z) = \left[h(h-1) + \bar h(\bar h - 1)\right]\Psi_{h,\bar h}(z,\bar z)$$
$$\mathscr{D}_z = z^2(1-z)\frac{\partial^2}{\partial z^2} - (1 - h_{13} + h_{24})z^2\frac{\partial}{\partial z} + h_{13}h_{24}z$$
admits solutions of the form $$\Psi_{h,\bar h}(z,\bar z) = \Psi_h(z)\Psi_{\bar h}(\bar z)\,.$$

- Conformal partial waves $$\Psi_h(z) = \frac{1}{2}\left(Q(h)k_h(z) + Q(1-h)k_{1-h}(z)\right), \quad h = \frac{1}{2} + \alpha, \quad \alpha \in i\mathbb{R},$$
where $k_{h,\bar h}(z,\bar z) = k_h(z)k_{\bar h}(\bar z)$, $k_h(z) = z^h {}_2F_1(h - h_{12}, h + h_{34}; 2h; z)$ are $SL(2,\mathbb{C})$ blocks.

- $Q(h)$ fixed by **orthogonality** $$\langle \Psi_h, \Psi_{h'}\rangle = \int_0^1 \frac{dz}{\mu(z)}\Psi_h(z)\Psi_{h'}(z) = \frac{N(h)}{2}\left[\delta(h+h') + \delta(h-h')\right]$$

- **Completeness** $$\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{dh}{N(h)}\Psi_h(z)\Psi_h(z') = \mu(z)\delta(z-z'), \quad \mu(z) = z^2(1-z)^{h_{12}-h_{34}}$$

[Hogervorst, Rutter, van Rees '17, '20]

Conformal blocks

$$f_t(z,\bar z) = N_{gm}(\beta)\,|z|^2\,|1-z|^{h_{13}-h_{24}}\delta(z-\bar z) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{dh}{N(h)}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{d\bar h}{N(\bar h)}g(h,\bar h)\Psi_{h,\bar h}(z,\bar z)$$

- Upon deforming the contour

$$f_t(z,\bar z) = n(\beta)\Big[\sum_{n=1}^{\infty} D^{nn}C_{13n}C_{24n}\cos\pi\left(\frac{n}{2}+h_{13}\right)\cos\pi\left(\frac{n}{2}+h_{24}\right)k_{\frac{1+n}{2}}(z)k_{\frac{1+n}{2}}(\bar z)$$

$$+\frac{1}{2}\int_{-i\infty}^{i\infty} d\alpha\, C^L_{13\alpha}C^L_{24\alpha}D^{L,\alpha\alpha}k_{\frac{1}{2}+\alpha}(z)k_{\frac{1}{2}-\alpha}(\bar z)\Big]$$

[Atanasov, Melton, A.R., Strominger ’21]

Conformal blocks

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$$\langle f_t(z,\bar{z}), \Psi_{h,\bar{h}}(z,\bar{z})\rangle$$

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$$g(h,\bar h) \to \langle f_t(z,\bar z), \Psi_{h,\bar h}(z,\bar z)\rangle$$

- Upon deforming the contour

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$$+\frac{1}{2}\int_{-i\infty}^{i\infty} d\alpha C^L_{13\alpha}C^L_{24\alpha}D^{L,\alpha\alpha}k_{\frac{1}{2}+\alpha}(z)k_{\frac{1}{2}-\alpha}(\bar z)\Big]$$

- Massive scalar exchanges

$$C_{ijn} = \frac{g}{m^4}\left(\frac{m}{2}\right)^{2h_i+2h_j} \mathrm{B}\left(\frac{1+n}{2}+h_{ij}, \frac{1+n}{2}-h_{ij}\right), \quad D^{nn} = \frac{nm^2}{2\pi}.$$

C_{ijn}: matches 3-point of 2 massless and 1 massive scalars

D^{nn}: massive 2-point

[Atanasov, Melton, A.R., Strominger '21]

Conformal blocks

$$f_t(z,\bar{z}) = N_{gm}(\beta)|z|^2|1-z|^{h_{13}-h_{24}}\delta(z-\bar{z}) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{dh}{N(h)}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{d\bar{h}}{N(\bar{h})}g(h,\bar{h})\Psi_{h,\bar{h}}(z,\bar{z})$$

$$\langle f_t(z,\bar{z}), \Psi_{h,\bar{h}}(z,\bar{z})\rangle$$

- Upon deforming the contour

$$f_t(z,\bar{z}) = n(\beta)\Big[\sum_{n=1}^{\infty} D^{nn}C_{13n}C_{24n}\cos\pi\left(\frac{n}{2}+h_{13}\right)\cos\pi\left(\frac{n}{2}+h_{24}\right)k_{\frac{1+n}{2}}(z)k_{\frac{1+n}{2}}(\bar{z})$$

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- Massive light-ray exchanges

light-transformed massive 2-point

$$C^L_{ij\alpha} = -\pi i\frac{g}{m^4}\left(\frac{m}{2}\right)^{2h_i+2h_j}\frac{1}{\alpha}, \qquad D^{L,\alpha\alpha} = -\frac{m^2\alpha^2}{\pi^2 i}.$$

matches 3-point of 2 massless and
1 light-transformed massive scalars

[Atanasov, Melton, A.R., Strominger '21]

Summary

In general $$\widetilde{\mathscr{A}}(\beta, z) = I_{13-24} z^2 (1-z)^{h_{13}-h_{24}} \delta(z-\bar{z}) \sum_{n=0}^{\infty} \frac{c_n^{IR}(z)}{\beta - \beta_n} + \dots$$

- Poles at negative β capture information about low energy data.

- Analytic structure for positive β distinct in QFT and quantum gravity.

- Conformal block decomposition for residue with $c_n^{IR} = 1$ factorizes, massive conformal primary scalars and their light-transforms are exchanged.

Open questions

- What about all other residues?
- Bulk unitarity $\Longrightarrow$ positivity constraints built into c_n^{IR}; here z is fixed, constraints complementary to bulk dispersion relations?
- t-channel conformal block expansion appears to be positive for equal external dimensions $\Longrightarrow$ bootstrap?
- Contour deformation appears to relate UV and IR: need better understanding of asymptotics in β plane.
- Can celestial spectrum, OPEs + analytic structure in β determine 4-point scattering amplitudes?