Replicas & RG:
Case study of Random Field Ising

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finite-T Ising in d-dim
w/ quenched disorder

Without dis

a) $\mathbb{Z}^d$ lattice

b) $s_i = \pm 1$

c) $H = -\sum_{\text{nearest}} J s_i s_j$

Bond disorder

$$H \to -\sum (J + \delta J_{ij}) s_i s_j$$

$$\langle \delta J_{ij} \rangle = 0$$

Field dis

$$H \to -\sum J s_i s_j + \sum_i h_i s_i$$

$$\langle h_i \rangle = 0 \qquad \langle h_i^2 \rangle = H$$

Quenched

$$\overline{\langle s_i s_j \rangle} = \overline{Z[h]^{-1} \mathrm{Tr}\, s_i s_j e^{-H[h_i]}}$$

average over h.

W/out disorder

$$\mathcal{L} = (\partial\varphi)^2 + V(\varphi) \qquad V = m^2\phi^2 + \lambda\phi^4$$

$\uparrow$ tune

Disorder : $\int dx\, j(x)\, \mathcal{O}(x)$ $\qquad \overline{j(x)\, j(x')} = H\delta(x-x')$

$\mathcal{O}(x) = \phi^2$ bond

$= \phi$ field.

Method of replicas

a) $S \longrightarrow S_1 + \dots + S_n$

$$= \sum_{i=1}^{n} (\partial\varphi_i)^2 + V(\phi_i) + \int j(x) \sum \mathcal{O}_i(x)$$

$n \to 0$

$-H\left(\sum \mathcal{O}_i\right)^2$

b) $\int \mathcal{D}j\; e^{-\int j^2/2H\, d^2x}$ | c)

Look $O_i \, O_j$ $\quad i \neq j$

$\Delta(O) < \frac{d}{2}$ — dis. rel.

Bond: $-H \left(\phi_1^2 + \dots + \phi_n^2\right)^2$

- $S_n \ltimes (\mathbb{Z}_2)^n$ $\quad (\phi_i \to -\phi_i)$
- weakly rel. in $d = 4 - \epsilon$
  - upper crit. dim $d_{uc} = 4$.
  - CFT (n)

2d $\quad \varepsilon_i \, \varepsilon_j$

$[\varepsilon] = 1$

Field

$$S' = \underbrace{\sum (\partial \varphi_i)^2}_{} + V(\phi_i) - H\big(\underbrace{\sum \phi_i}_{\to\ \omega}\big)^2$$

$\lambda \phi^4$ ↑ strongly rel.

- $\mathbb{Z}_2 \times S_n$
-
- $d_{uc} = 6$
- no CFT for $\forall n$

$\phi_1 = \varphi + \omega/2$
$+ (n-1)(\varphi - \omega/2)$
$+ \cancel{\sum \chi_i}$
$= n \cdot \varphi + \omega$

$[\lambda] = 4 - d$
$[H] = 2$

$\lambda_{eff} = \lambda H \Rightarrow [\lambda_{eff}] = 6 - d$

Cardy 1985

$$\begin{cases} \phi_1 = \varphi + \omega/2 \\ \phi_i = \varphi - \omega/2 + \chi_i & i = 2 \dots n \end{cases}$$

$$\sum \chi_i = 0$$

$$\text{Kin}: \quad \boxed{\underbrace{\partial\varphi\,\partial\omega - \frac{H}{2}\omega^2}_{} + (\partial\varphi)^2 \times O(n)} + \sum(\partial\chi_i)^{\sim} + \boxed{O(n)}$$

$$\Delta_\varphi = \frac{d}{2} - 2 \qquad \Delta_\omega = \frac{d}{2} \qquad \Delta = \frac{d}{2} - 1$$

$$\text{Cardy}\left[\sum(\phi_i)^2\right] = \omega\varphi + \sum\chi_i^2$$

$$\text{Cardy}\left[\underbrace{\sum(\phi_i)^4}\right] = \boxed{\underbrace{\omega\varphi^3 + \varphi^2\sum\chi_i^2}_{}} + \ldots$$

marginal in $d=6$ irrel.

S_n - singlets $\qquad \sum_{i=1}^{n}(\phi_i)^4$

φ_1

$$\langle \varphi_1 \ \varphi_2 \rangle \neq 0$$

$$\langle \text{singlet} \ \ \text{singlet} \ldots \rangle = 0.$$

Parisi – Sourlas SUSY

$n-1$ $\quad x_i \quad \sum x_i = 0 \quad -2$ fields

$$x_i \to \psi, \bar\psi$$

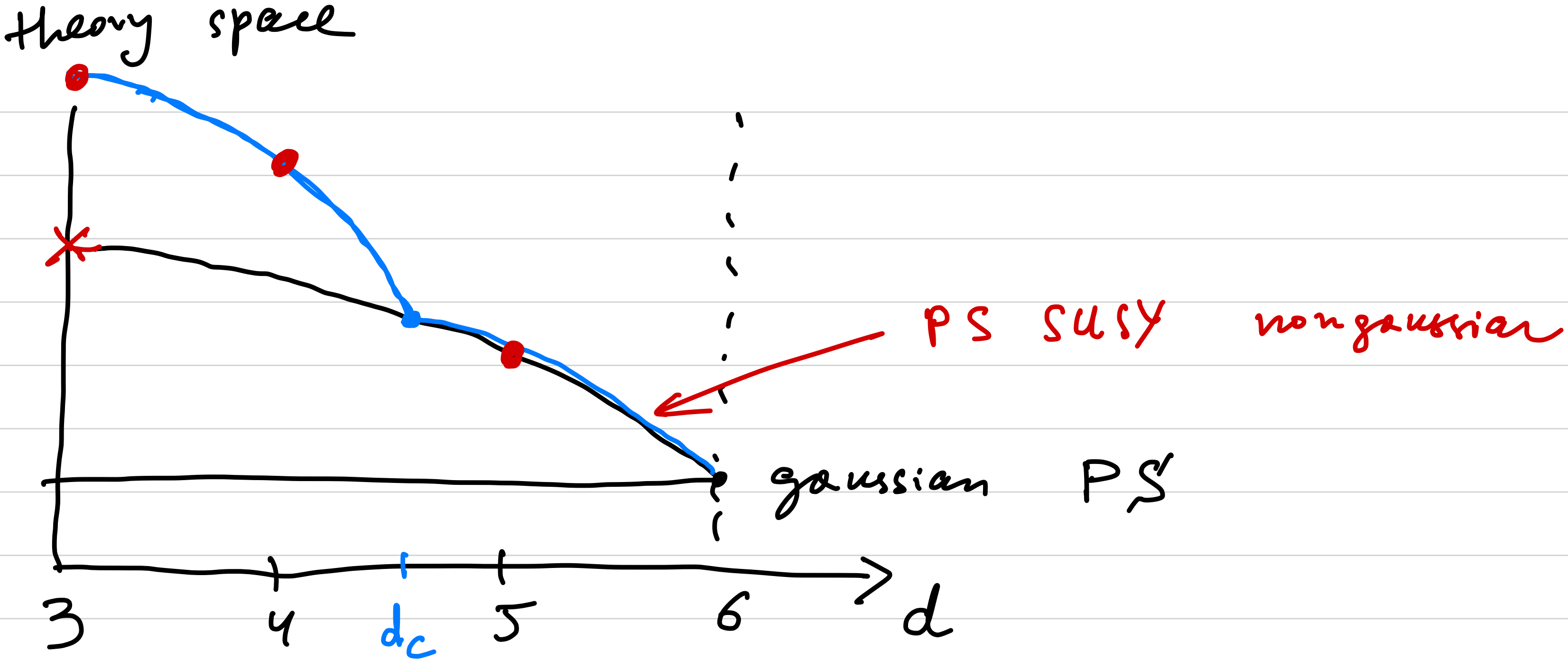
$$\to \partial\varphi\,\partial\omega - \omega^2 + \partial\psi\,\partial\bar\psi + m^2(\omega\varphi + \psi\bar\psi) + \lambda(\omega\varphi^3 + \varphi^2\psi\bar\psi)$$

$$\Phi(x,\theta,\bar\theta) = \varphi(x) + \theta\bar\psi + \bar\theta\psi + \theta\bar\theta\,\omega$$

$$\int d^dx\, d\theta\, d\bar\theta$$

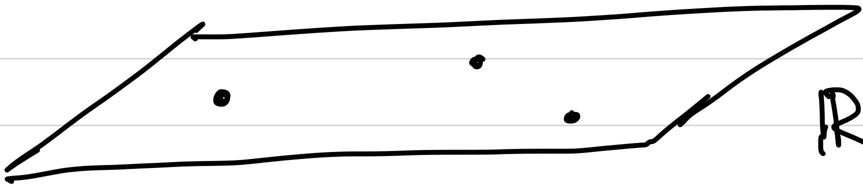
$Osp(2|2)$

$$\int d^{d-2}x\left[(\partial\varphi)^2 + V(\phi)\right]$$ "dim. reduction"



$OSp(2|2)$

$$\theta = \bar{\theta} = 0$$
$$x_\perp = 0$$



$\mathbb{R}^{d-2} \subset \mathbb{R}^{d|2}$

$$\Phi(x, \theta, \bar{\theta})$$

$$\begin{cases} \left(\sum x_i^2\right)^2 & \text{— susy-null} \qquad (\psi\bar{\psi})^2 = 0 \\ \left(\underbrace{\sum x_i^3}\right)^2 - \tfrac{3}{2}\left(\sum x_i^2\right)\left(\underbrace{\sum x_i^4}\right) & \text{— non-susy-writable} \end{cases}$$

susy-writable

$$(\varphi, \omega, x_i) \qquad O(n-2)$$

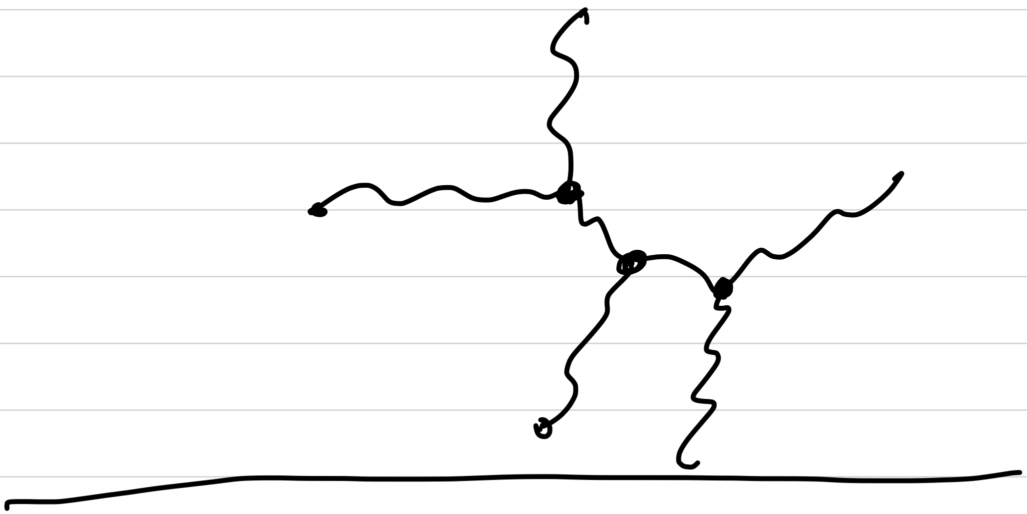
$$\sum x_i^2 \to \psi\bar{\psi}$$

$$S_n \ltimes \mathbb{Z}_2^n \longleftrightarrow S_n \cancel{\ltimes} \mathbb{Z}_2$$

only @ $n=0$

$$(\partial\varphi)^2 + h\varphi + \lambda\varphi^3$$

$d=8$

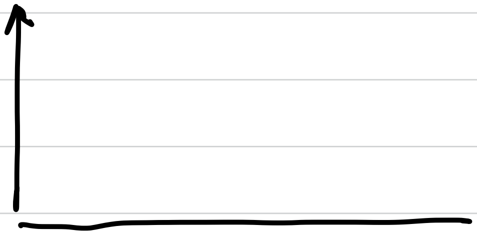


$(x^2)^2$

$$8 - 2\epsilon - \frac{2}{27}\epsilon^2 + \dots$$

dynamical crit phen.

t



$d+1$

z

CFT_d