Progress in Celestial Holography

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2104.13432 with A. Atanasov, W. Melton and A. Strominger
Motivation

• What are the observables of quantum gravity?

1. Effective field theory perspective: at scales $E \ll M_{Pl}$

$$\sim \sum_{r \leq m} (G_N s)^m a_r(s, t) \log^r \left( \frac{s}{M_{Pl}^2} \right) + \ldots$$

• perturbatively calculable;

• predictive, contain information about deflections, scattering angles, etc.
Motivation

• What are the observables of quantum gravity?

2. For $E \sim M_{pl}$ we don’t know...

• At high enough center of mass energies black holes can form and evaporate; perturbation theory breaks down.
Motivation

• What are the observables of quantum gravity?

2. Answer in AdS quantum gravity provided by holography:

Observables are encoded in correlation functions of a conformal field theory on the boundary.

Many insights: black hole microstates, bulk unitarity/locality/causality, spacetime emergence, ...
Motivation

• What about asymptotically flat quantum gravity?

\[ S_{BH} = \frac{A}{4G_N} \]

suggests holography should again give the answer, but:

• Boundaries are null - hard to formulate a quantum field theory on \( \mathcal{I} \).

• Data at \( \mathcal{I}^+ \) and \( \mathcal{I}^- \) are related by S-matrix; moreover, data on \( \mathcal{I}^+ \) and \( \mathcal{I}^- \) are constrained.
Motivation

• What about asymptotically flat quantum gravity?

\[ S_{BH} = \frac{A}{4G_N} \]
suggests holography should again give the answer, but:

• Radiation \( \rightarrow \) conserved quantities are subtle to define;

[Lee, Iyer, Wald, Zoupas '90s]

• Infinite dimensional asymptotic symmetry group \( \rightarrow \) S-matrix ill-defined, \( |out\rangle = S|in\rangle \) up to supertranslations. [Bondi, van der Burg, Metzner, Sachs '62]
Asymptotic symmetries

- Ambiguity fixed by matching conditions. [Strominger '13]

- Eg. supertranslation charges are antipodally matched \( Q^+ = Q^- \).

- Their conservation \( \langle p_f | [Q, \mathcal{S}] | p_i \rangle = 0 \) implies the soft graviton theorem!

**Soft theorems:** [Weinberg '65]
Asymptotic symmetries

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Soft theorems:
[Weinberg ’65]
Asymptotic symmetries

- Ambiguity fixed by matching conditions. [Strominger ’13]

- Subleading soft theorems → more symmetries? YES!

- Subleading soft graviton theorem → infinite dimensional enhancement of Lorentz symmetry [Kapec, Lysov, Pasterski, Strominger ’16]

- Associated charge conservation implies

\[
\langle T_{zz} O_1(\omega_1, z_1, \bar{z}_1) \ldots \rangle = \sum_{i=1}^{n} \left[ \frac{\hat{h}_i}{(z - z_i)^2} + \frac{\partial_i}{z - z_i} \right] \langle O_1(\omega_1, z_1, \bar{z}_1) \ldots \rangle,
\]

\[
\hat{h}_i = \frac{1}{2} (s_i - \omega_i \partial \omega_i).
\]

[Kapec, Mitra, A.R., Strominger ’16]
Celestial holography

\[ \langle T_{zz} O_1(\omega_1, z_1, \bar{z}_1) \ldots \rangle = \sum_{i=1}^{n} \left[ \frac{\hat{h}_i}{(z - z_i)^2} + \frac{\partial_i}{z - z_i} \right] \langle O_1(\omega_1, z_1, \bar{z}_1) \ldots \rangle, \]

\[ \hat{h}_i = \frac{1}{2} (s_i - \omega_i \partial \omega_i). \]

- Momentum eigenstates don’t diagonalize \( \hat{h}_i \), but boost eigenstates do!
- Study scattering in basis of boost eigenstates.
- Celestial amplitudes = S-matrices in boost basis
- For asymptotic massless scattering states:

\[ |p_i\rangle \rightarrow |\omega_i, z_i, \bar{z}_i\rangle \rightarrow |\Delta_i, z_i, \bar{z}_i\rangle = \int_0^{\infty} d\omega_i \omega_i^{\Delta_i-1} |\omega_i, z_i, \bar{z}_i\rangle. \]
Novel Features of CCFT

• Lorentz symmetry → $SL(2,\mathbb{C})$ covariance  
  [Pasterski, Shao, Strominger '17]

• Translations relate celestial amplitudes with shifted weights.  
  [Donnay, Puhm, Strominger '18; Stieberger, Taylor '18; Law, Zlotnikov '19]

• Soft symmetries associated with conformally soft operators of negative half-integer dimensions and their canonical conjugate Goldstone modes.  
  [Pate, A.R., Strominger '19; Donnay, Pasterski, Puhm '20; Guevara, Himwich, Pate, Strominger, Strominger '21]

• Soft theorems constrain leading, subleading OPE coefficients in EYM.  
  [Pate, A.R., Strominger, Yuan '19; Banerjee, Ghosh '20; Ebert, Sharma, Wang '20]

• IR divergences are captured by vertex operators of Goldstones.  
  [Nande, Pate, Strominger '17; Himwich, Narayanan, Paul, Pate, Strominger '20]
Outline

Can we learn anything about quantum gravity?

1. Analytic structure of celestial 4-point scattering in the complex boost weight $\beta$-plane captures perturbative and non-perturbative aspects of bulk scattering.

   [Arkani-Hamed, Pate, A.R., Strominger '20]

2. Conformal block expansion provides evidence that residues of poles in $\beta$ can be reconstructed from celestial OPE data.

   [Atanasov, Melton, A.R., Strominger '21]
Massless scalar 4-point scattering

\[ \mathbb{A}(p_i) = \mathcal{M}(s, t)\delta^{(4)} \left( \sum_{i=1}^{4} p_i \right) \]

\[ s = - (p_1 + p_2)^2 \equiv \omega^2 \]

\[ t = - (p_1 + p_3)^2 \equiv -z\omega^2 \]

\[ \tilde{\mathbb{A}}(z_i, \bar{z}_i; \beta) = K(z_i, \bar{z}_i)X(z, \beta) \int_{0}^{\infty} d\omega \omega^{\beta-1} \mathcal{M}(\omega^2, -z\omega^2) \]

\[ \beta = \sum_{i=1}^{4} \Delta_i - 4, \quad z = -t/s = \frac{z_{13}z_{24}}{z_{12}z_{34}} \in [0,1] \]

\[ K(z_i, \bar{z}_i) = \prod_{i<j}^{4} \frac{z_{ij}^{\frac{h_i - h_j}{3}} - \bar{z}_{ij}^{\frac{\bar{h}_i - \bar{h}_j}{3}}}{z_{ij}^{\frac{\bar{h}_i - \bar{h}_j}{3}} - \bar{z}_{ij}^{\frac{h_i - h_j}{3}}} \]

\[ X(z, \beta) = \delta(z - \bar{z}) |z(1-z)|^{\frac{1}{6}(\beta+4)} \]
Part I: Analytic structure in $\beta$
Imprints of UV physics

• General formula:

\[ \widetilde{\mathcal{A}}(z_i, \bar{z}_i; \beta) = K(z_i, \bar{z}_i)X(z, \beta) \int_0^\infty d\omega \omega^{\beta-1} \mathcal{M}(\omega^2, -z\omega^2) \equiv \mathcal{A}(\beta, z), \text{ dynamics} \]

\[ \text{kinematics} \]

• Consider the (sick) example:

\[ \mathcal{M} \propto \omega^p \implies \mathcal{A} \propto \int_0^\infty d\omega \omega^{\beta+p-1} \propto \delta(\beta + p), \beta + p \in i\mathbb{R} \]

Scattering amplitudes with poor UV behavior
\[ \implies \text{badly behaved, non-analytic celestial amplitudes.} \]
Analytic structure in $\beta$

- General formula:

\[
\overline{\mathcal{A}}(z_i, \bar{z}_i; \beta) = K(z_i, \bar{z}_i)X(z, \beta) \int_0^\infty d\omega \omega^{\beta-1} \mathcal{M}(\omega^2, -z\omega^2).
\]

\[
\mathcal{M} = \lambda \frac{M^2}{\omega^2 - M^2} \quad \Rightarrow \quad \mathcal{A} \propto \frac{\lambda M^\beta}{\sin \pi \beta/2}.
\]

- $\mathcal{A}(\beta, z)$ is well defined provided $\mathcal{M}$ falls off fast enough as $\omega \to \infty$.

  eg. s-channel pole

  \[
  \mathcal{M} = \lambda \frac{M^2}{\omega^2 - M^2} \quad \Rightarrow \quad \mathcal{A} \propto \frac{\lambda M^\beta}{\sin \pi \beta/2}.
  \]

- Poles in $\omega$ $\quad \Rightarrow \quad$ infinite sequence of poles in $\beta$-plane!
Analytic structure in $\beta$

- Low-energy expansion with massive states integrated out and no massless loops

$$\mathcal{M}(s, t) = \sum_{\Delta, q} a_{\Delta, q} s^{\Delta - q} t^q \iff \mathcal{M}(\omega^2, -z\omega^2) = \sum_{\Delta, q} a_{\Delta, q} \omega^{2\Delta} (-z)^q .$$

leads to celestial amplitude (with cutoff $\omega_*$)

$$\mathcal{A}(\beta, z) = \sum_{\Delta} \frac{\tilde{a}^{lR}(z)}{\beta + 2\Delta} + \ldots$$

- First pole in $\beta$ determined by lowest dimension operators in EFT.

- Sequence of poles at increasingly negative even integer $\beta$ carries information about higher dimension operators; residues obey positivity constraints.

[Arkani-Hamed, Huang, Huang '20]
Analytic structure in $\beta$

• If the scattering amplitude also admits an expansion around

$$\omega \rightarrow \infty \quad \Longrightarrow \quad A(\beta, z) = \sum_{n \in \mathbb{Z}_-} \frac{\tilde{a}_n(UV(n))}{\beta + 2n} + \ldots$$

• Accounting for loop effects

$$M(\omega^2, -z\omega^2) = \sum_{m,n,r \leq m} a_{m,n,r}(z)(G_N\omega^2)^m \omega^{2n} \log^r \frac{\omega}{\Lambda_{\text{UV}}}$$

yields higher order poles in $\beta$

$$A(\beta, z) \supset \int_0^{\omega_*} d\omega \omega^{\beta-1} \log^r \omega \propto \frac{\partial^r}{\partial \beta^r} \frac{1}{\beta} \propto \frac{1}{\beta^{r+1}}.$$
Another UV constraint

• Exponential fall-off in the hard scattering limit

\[
\lim_{\omega \to \infty} \mathcal{M}(\omega^2, -z\omega^2) \propto e^{-\omega^2/M^2}.
\]

• Large $\beta$ limit localizes the integrand at high energy and

\[
\lim_{\beta \to \infty} \mathcal{A}(\beta, z) \to \int_0^\infty d\omega \omega^{\beta-1} e^{-\omega^2/M^2} = \frac{M^\beta}{2} \Gamma(\beta/2).
\]

• Examples: black holes $S_{BH} = 4\pi G_N \omega^2$, $M \propto M_{Pl}$;

  tree-level string amplitudes $M \propto M_s$.

• Absence of poles in the right complex $\beta$ plane is a sharp signature of quantum gravity!
Part II: Analytic structure in $\mathbb{Z}$
Analytic structure in $\mathcal{Z}$

- Tree-level, t-channel scattering
  \[
  \mathcal{M}(s, t) = -g^2 \frac{1}{t - m^2} \rightarrow \\
  \widetilde{A}(z_i, \bar{z}_i; \beta) = g^2 K(z_i, \bar{z}_i)X(z, \beta) \int_0^\infty d\omega \omega^{\beta-1} \frac{1}{z\omega^2 + m^2} \\
  = I_{13-24}(z_i, \bar{z}_i) N_g m(\beta) |z|^2 |1 - z|^{h_{13} - h_{24}} \delta(z - \bar{z})
  \]

Goal: Decompose $f_t(z, \bar{z})$ into conformal blocks.
Analytic structure in $z$

- Tree-level, t-channel scattering

$$\mathcal{M}(s, t) = -g^2 \frac{1}{t - m^2} \rightarrow$$

$$\widetilde{\mathcal{A}}(z_i, \bar{z}_i; \beta) = I_{13-24}(z_i, \bar{z}_i)N_{gm}(\beta) |z|^2 |1 - z|^{h_{13} - h_{24}} \delta(z - \bar{z}) f_t(z, \bar{z})$$

Roadmap: Treat $z, \bar{z}$ as real independent variables and expand $f_t(z, \bar{z})$ on a complete set of orthogonal solutions to the two particle conformal Casimir over the Lorentzian square, $z, \bar{z} \in [0,1]$. 
Conformal partial waves

• 1-3 conformal Casimir equation
  \[(\mathcal{D}_z + \mathcal{D}_\bar{z}) \Psi_{h,\bar{h}}(z, \bar{z}) = [h(h - 1) + \bar{h}(\bar{h} - 1)] \Psi_{h,\bar{h}}(z, \bar{z})\]
  \[\mathcal{D}_z = z^2(1 - z) \frac{\partial^2}{\partial z^2} - (1 - h_{13} + h_{24})z^2 \frac{\partial}{\partial z} + h_{13}h_{24}z\]

  admits solutions of the form
  \[\Psi_{h,\bar{h}}(z, \bar{z}) = \Psi_h(z)\Psi_{\bar{h}}(\bar{z}).\]

• Conformal partial waves
  \[\Psi_h(z) = \frac{1}{2} \left( Q(h)k_h(z) + Q(1 - h)k_{1-h}(z) \right), \quad h = \frac{1}{2} + \alpha, \quad \alpha \in i\mathbb{R},\]
  where
  \[k_{h,\bar{h}}(z, \bar{z}) = k_h(z)k_{\bar{h}}(\bar{z}), \quad k_h(z) = z^{h_{12}}F_1(h - h_{12}, h + h_{34}; 2h; z)\]
  are SL(2, \mathbb{C}) blocks.

• Q(h) fixed by orthogonality
  \[\langle \Psi_h, \Psi_{h'} \rangle = \int_0^{1} \frac{dz}{\mu(z)} \Psi_h(z)\Psi_{h'}(z) = \frac{N(h)}{2} \left[ \delta(h + h') + \delta(h - h') \right]\]

• Completeness
  \[\int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{dh}{N(h)} \Psi_h(z)\Psi_{h'}(z') = \mu(z)\delta(z - z'), \quad \mu(z) = z^2(1 - z)^{h_{12} - h_{34}}\]

[Hogervorst, Rutter, van Rees '17, '20]
Conformal blocks

\[
 f_t(z, \bar{z}) = N_{gm}(\beta) \left| \frac{z}{1 - z} \right|^2 \left| z \right|^{h_{13} - h_{24}} \delta(z - \bar{z}) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{dh}{N(h)} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\bar{h}}{N(\bar{h})} g(h, \bar{h}) \Psi_{h, \bar{h}}(z, \bar{z})
\]

- Upon deforming the contour

\[
 f_t(z, \bar{z}) = n(\beta) \left[ \sum_{n=1}^{\infty} D^{nn} C_{13n} C_{24n} \cos \pi \left( \frac{n}{2} + h_{13} \right) \cos \pi \left( \frac{n}{2} + h_{24} \right) k_{\frac{1+n}{2}}(z) k_{\frac{1+n}{2}}(\bar{z}) + \frac{1}{2} \int_{-i\infty}^{i\infty} d\alpha C_{13\alpha} C_{24\alpha} D^{L, \alpha \alpha} k_{\frac{1+\alpha}{2}}(z) k_{\frac{1-\alpha}{2}}(\bar{z}) \right]
\]

[Atanasov, Melton, A.R., Strominger '21]
Conformal blocks

\[ f_t(z, \bar{z}) = N_g m(\beta) |z|^2 |1 - z|^{h_{13} - h_{24}} \delta(z - \bar{z}) = \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{dh}{N(h)} \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{d\bar{h}}{N(\bar{h})} g(h, \bar{h}) \Psi_{h, \bar{h}}(z, \bar{z}) \]

\[ \langle f_t(z, \bar{z}), \Psi_{h, \bar{h}}(z, \bar{z}) \rangle \]

- Upon deforming the contour

\[ f_t(z, \bar{z}) = n(\beta) \left[ \sum_{n=1}^{\infty} D_{13n} C_{24n} \cos \pi \left( \frac{n}{2} + h_{13} \right) \cos \pi \left( \frac{n}{2} + h_{24} \right) k_{\frac{1}{2} + n}(z) k_{\frac{1}{2} + n}(\bar{z}) \right. 
\left. + \frac{1}{2} \int_{-i\infty}^{i\infty} d\alpha C_{13\alpha} C_{24\alpha} D_{L, \alpha} k_{\frac{1}{2} + \alpha}(z) k_{\frac{1}{2} - \alpha}(\bar{z}) \right] \]

[Atanasov, Melton, A.R., Strominger '21]
Conformal blocks

\[ f_t(z, \bar{z}) = N_g m(\beta) |z|^2 |1 - z|^{h_{13} - h_{24}} \delta(z - \bar{z}) = \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{dh}{N(h)} \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{d\bar{h}}{N(\bar{h})} g(h, \bar{h}) \Psi_{h, \bar{h}}(z, \bar{z}) \]

\[ \langle f_t(z, \bar{z}), \Psi_{h, \bar{h}}(z, \bar{z}) \rangle \]

• Upon deforming the contour

\[ f_t(z, \bar{z}) = n(\beta) \left[ \sum_{n=1}^{\infty} D^{mn} C_{13n} C_{24n} \cos \pi \left( \frac{n}{2} + h_{13} \right) \cos \pi \left( \frac{n}{2} + h_{24} \right) k_{\frac{1}{2} + n} k_{\frac{1}{2} + n} \right] \]

\[ + \frac{1}{2} \int_{-i\infty}^{i\infty} d\alpha C^L_{13\alpha} C^L_{24\alpha} D^{L,\alpha\alpha} k_{\frac{1}{2} + \alpha} k_{\frac{1}{2} + \alpha} \]

• Massive scalar exchanges

\[ C_{ij} = \frac{g}{m^4} \left( \frac{m}{2} \right)^{2h_i + 2h_j} B \left( \frac{1 + n}{2} + h_{ij}, \frac{1 + n}{2} - h_{ij} \right), \quad D^{mn} = \frac{nm^2}{2\pi} \]

matches 3-point of 2 massless and 1 massive scalars

[Atanasov, Melton, A.R., Strominger '21]
Conformal blocks

\[ f_t(z, \bar{z}) = N_{gm}(\beta) |z|^2 |1 - z|^{h_{13} - h_{24}} \delta(z - \bar{z}) = \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{dh}{N(h)} \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{d\bar{h}}{N(\bar{h})} g(h, \bar{h}) \Psi_{h, \bar{h}}(z, \bar{z}) \]

\[ \langle f_t(z, \bar{z}), \Psi_{h, \bar{h}}(z, \bar{z}) \rangle \]

- Upon deforming the contour

\[ f_t(z, \bar{z}) = n(\beta) \left[ \sum_{n=1}^{\infty} D^{mn} C_{13n} C_{24n} \cos \pi \left( \frac{n}{2} + h_{13} \right) \cos \pi \left( \frac{n}{2} + h_{24} \right) k_{\frac{1}{2} + n}(z) k_{\frac{1}{2} + n}(\bar{z}) \right. \]

\[ + \frac{1}{2} \int_{-i\infty}^{i\infty} d\alpha C_{13\alpha} C_{24\alpha} D^{L,\alpha\alpha} k_{\frac{1}{2} + \alpha}(z) k_{\frac{1}{2} - \alpha}(\bar{z}) \]

- Massive light-ray exchanges

\[ C_{ij\alpha}^L = -\pi i \frac{g}{m^4} \left( \frac{m}{2} \right)^{2h_i + 2h_j} \frac{1}{\alpha}, \quad D_{L,\alpha\alpha} = -\frac{m^2 \alpha^2}{\pi^2 i}. \]

matches 3-point of 2 massless and 1 light-transformed massive scalars

light-transformed massive 2-point

[Atanasov, Melton, A.R., Strominger '21]
Summary

In general

\[ \widehat{A}(\beta, z) = I_{13-24}z^2(1 - z)^{h_{13} - h_{24}}\delta(z - \bar{z}) \sum_{n=0}^{\infty} \frac{c_n^{IR}(z)}{\beta - \beta_n} + \ldots \]

- Poles at negative \( \beta \) capture information about low energy data.

- Analytic structure for positive \( \beta \) distinct in QFT and quantum gravity.

- Conformal block decomposition for residue with \( c_n^{IR} = 1 \) factorizes, massive conformal primary scalars and their light-transforms are exchanged.
Open questions

• What about all other residues?

• Bulk unitarity $\implies$ positivity constraints built into $c_{n}^{IR}$; here $z$ is fixed, constraints complementary to bulk dispersion relations?

• t-channel conformal block expansion appears to be positive for equal external dimensions $\implies$ bootstrap?

• Contour deformation appears to relate UV and IR: need better understanding of asymptotics in $\beta$ plane.

• Can celestial spectrum, OPEs + analytic structure in $\beta$ determine 4-point scattering amplitudes?