Microscopic Entropy of AdS Black Holes

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Microscopics of Black Hole Entropy

- The Bekenstein-Hawking **area law** for black hole entropy:

$$S = \frac{A}{4G_N} \ .$$

- In **favorable cases** string theory offers a **microscopic** interpretation of the black hole: specific constituents, ...

- **Statistical** understanding $S = \ln \Omega$ of the area law and more: **higher dimension** operators, **quantum** corrections, ...

- These developments are among the **most prominent successes** of string theory as a **theory of quantum gravity**.

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AdS_5 Holography

- The best studied example of holography: **String theory on $\text{AdS}_5 \times S^5$** is dual to **N=4 SYM** in $D = 4$.

- **Microsopic details** well understood (Quantum Field Theory!)

- The **area law** entropy of black holes in AdS_5 is a **crude target**: just the asymptotic density of states.

- Yet: only recently were **quantitative agreements established** in this context.

Hosseini, Hristov, and Zaffaroni 1705.05383
Cabo-Bizet, Cassani, Martelli, and Murthy 1810.11442
Choi, Kim, Kim, and Nahmgoong 1810.12067
Benini and Milan 1811.04017
Zaffaroni (review) 1902.07176.

This Talk

Overall focus:
Supersymmetric AdS_5 black holes and their **nearBPS** relatives.

Outline:

- Black hole **thermodynamics**: "phenomenology".
- Lessons from black holes in **AdS_3**.
- Relation to **$nAdS_2/CFT_1$ correspondence**.
- **Structure** of microscopic theory $\Rightarrow$ some **puzzles**.

Ongoing research (supported by DoE) with Sangmin Choi, Nizar Ezroura, Junho Hong, Siuyl Lee, Billy Liu, Jim Liu, Jun Nian, Shruti Paranjape, Yangwenxiao Zeng.

Quantum Numbers

- Geometry: $AdS_5 \times S^5$ has (superconformal extension of) $SO(2,4) \times SO(6)$ symmetry.
- Fields in $SO(2,4)$ representations:
  conformal weight E and **angular momenta** $J_{a,b}$.
- Fields in $SO(6)$ representations: **R-charges** Q_I $(I = 1, 2, 3)$.
- So asymptotic data of (electric) black holes in AdS_5:
  Mass M**, Angular momenta** $J_{a,b}$**, and** 3 **R-charges** Q_I.

Classical Black Hole Solutions

- General supergravity solution (Wu '11) .
  Independent mass M, angular momenta $J_{a,b}$, R-charges Q_I.
  Not widely known (and exceptionally complicated).

- General BPS (**supersymmetric**) solution: Gutowski+Reall '05.

- BPS mass = **"ground state energy"** ($g = \ell_5^{-1}$):

$$M = \sum_I Q_I + g(J_a + J_b)$$

Novel features (not shared by asymptotically flat black holes):

- Only 2 SUSY's preserved = $\frac{1}{16}$ **of maximal**.

- Quantum numbers Q_I, J_a, J_b are related by a **nonlinear constraint**. Specifically, **rotation is mandatory**.

The Black Hole Entropy (BPS limit)

$$S = 2\pi\sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3 - \frac{1}{2}N^2(J_a + J_b)}$$

- $Q_I, J_{a,b} =$ **integral** charges. $N =$**rank** of dual gauge group.
- There are two scales in the problem: $g = \ell_5^{-1}$ and G_5 .
- They are **related as** $\frac{\pi}{4G_5}\ell_5^3 = \frac{1}{2}N^2$.
- **Classical charges** are $\sim N^2$ so the entropy is also $\sim N^2$.

The Constraint on Conserved Charges

BPS black holes all have charges satisfying:

$$h \equiv \left((Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3) - \frac{1}{2} N^2 (J_a + J_b) \right) \left(\frac{1}{2} N^2 + (Q_1 + Q_2 + Q_3) \right) - \frac{1}{2} N^2 J_a J_b + Q_1 Q_2 Q_3 = 0$$

Corollary: **two** distinct deformations break supersymmetry

- Recall: $\underbrace{T = 0}_{\textbf{extremality}} \quad \Leftrightarrow \quad \underbrace{M = M_{\rm ext}}_{\rm lowest\ mass\ (given\ conserved\ charges)}$.
- Standard SUSY breaking: **mass exceeds** $M_{\rm ext}$.
  Description: **raise the temperature** $\Rightarrow \quad T > 0$.
- **Alternative**: violate **constraint** by **adjusting conserved charges** while preserving $T = 0$ (retain $M = M_{\rm ext}$).

Constraint Follows from Supersymmetry

- (Inaccurate) lore: constraint required to avoid **naked closed timelike curves**.

- SUSY algebra + **unitarity** gives **BPS bound**:

$$\{Q, Q^\dagger\} = M - \underbrace{M_{\rm BPS}}_{\sum_I Q_I + g(J_a + J_b)} \geq 0$$

- Mass M of all black hole solutions satisfies identity with form

$$M - M_{\rm BPS} = \underbrace{(\ldots)^2}_{\equiv 0 \text{ for } T=0} + (\ldots)^2 \geq 0$$

BPS **saturation** shows 2nd $(\ldots)^2 = 0 \quad \Rightarrow \quad$ **constraint**.

- Constraint follows from **BPS** with **no other assumptions**.

Detour: BTZ Black Holes

Example: black holes in $AdS_3 \times S^3$ dual to CFT_2 with $(4,4)$ SUSY.

Analysis in AdS_3 spacetime and in CFT_2 are **very similar**.

Four quantum numbers: ϵ, p (AdS_3), j_R, j_L (S^3).
Conformal weights $h_{R,L} = \frac{1}{2}(\epsilon \pm p) + \frac{1}{4}k_{R,L}$ and **R-charges** $j_{R,L}$.

Partition function:

$$Z = \mathrm{Tr}\ e^{2\pi i\tau(L_0 - \frac{1}{4}k_R) + 2\pi i z j_R - 2\pi i\bar{\tau}(\bar{L}_0 - \frac{1}{4}k_L) + 2\pi i\bar{z}j_L}$$

$SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ invariant NS-**vacuum** $L_0, \bar{L}_0 \to 0$ controlled by Casimir energy.

BTZ Black Holes: Statistical Description

Modular transform $\tau \to -\tau^{-1}$ maps vacuum to statistical regime:

$$\ln Z = \frac{\pi i k_R}{2\tau}\left(1 - 4z^2\right) - \frac{\pi i k_L}{2\bar{\tau}}\left(1 - 4\bar{z}^2\right)$$

Legendre transform gives the correct black hole entropy:

$$S = 2\pi\sqrt{k_R(E+P) - \frac{1}{4}J_R^2} + 2\pi\sqrt{k_L(E-P) - \frac{1}{4}J_L^2}$$

Extremality ($T = 0$): $\frac{1}{2}(E-P) = \frac{1}{4k_L}J_L^2$

BPS saturation (chiral primary):

$$\underbrace{\frac{1}{2}(E-P) + \frac{1}{4}k_L}_{\text{conformal weight } h_L} = \frac{1}{2}J_L \quad \underset{\text{consistency}}{\Rightarrow} \quad \boxed{J_L = k_L}$$

2nd condition

Perspectives on Constraint from AdS_3/CFT_2

- Supersymmetric **states** in CFT_2:
  Chiral primaries $\bar{h} = \frac{1}{2} j_L$ with $0 \leq j_L \leq 2k_L$ (unitarity).

- Supersymmetric black hole **geometries**:
  Exist only for $J_L = k_L$ (so **two conditions** on parameters).

- **Elliptic genus**: index entirely **holomorphic** $\Rightarrow$ co-dimension 2 in parameter space. Inserting $(-)^F$ "averages" over all j_L.

Physics lesson: the **constraint emerges** from supersymmetry of the **ensemble average**.

Another Perspective: SUSY Breaking Mechanisms

$$S = 2\pi\sqrt{k_R(E+P) - \frac{1}{4}J_R^2} + \underbrace{2\pi\sqrt{k_L(E-P) - \frac{1}{4}J_L^2}}_{\text{SUSY breaking excitations}}$$

- **Conventional SUSY breaking** (temperature T):
  Activate excitations in the L sector.

- **Novel SUSY breaking**:
  L sector in **ground state** $\underbrace{\Rightarrow}_{J_L \neq k_L}$ $E > E_{\text{BPS}} = P + J_L - \frac{1}{2}k_L$.

  Additional excitations in the R sector.

Aside: 4D extremal Kerr breaks SUSY by the “novel” mechanism.

AdS_5 Black Hole: Heat Capacity

- Excite the black hole so mass above BPS bound

$$M = M_{\rm BPS} + \frac{1}{2}\left(\frac{C_T}{T}\right)\ T^2$$

C_T is the black hole **heat capacity** (proportional to T).

- Gravity computations give

$$\frac{C_T}{T} = \frac{8Q^3 + \frac{1}{4}N^4(J_1+J_2)}{\frac{1}{4}N^4 + \frac{1}{2}N^2(6Q - J_1 - J_2) + 12Q^2}$$

- Interpretation: **number of degrees of freedom** in **low energy excitions**.
$\frac{C_T}{T}$ analogous to the **central charge** $c_L = 6k_L$.

AdS$_5$ Black Hole: Capacitance

- BPS saturation implies the constraint so **no SUSY if the constraint is violated**.
- Then the **black hole mass exceeds the BPS bound**:

$$M_{\rm ext} = M_{\rm BPS} + \frac{1}{2}\left(\frac{C_\varphi}{T}\right)\left(\frac{\varphi}{2\pi}\right)^2 \ .$$

- C_φ is the **capacitance** of the black hole.
(The **potential** φ is defined precisely later)
- Gravity computations give

$$\frac{C_\varphi}{T} = \frac{8Q^3 + \frac{1}{4}N^4(J_1+J_2)}{\frac{1}{4}N^4 + \frac{1}{2}N^2(6Q^2 + J_1 + J_2) + 12Q^2}$$

- **Note**: $C_\varphi = C_T$. Excitations violating the constraint "cost" the same as those violating the extremality bound!

$\text{nAdS}_2/\text{CFT}_1$ Correspondence

- All BPS black holes have **AdS_2 near horizon geometry**.
- AdS_2 does not allow excitations (with finite energy): they always **deform the AdS_2 geometry**.
- This **strong IR dynamics** in two dimensions has a universal description in **effective quantum field theory**.
- There is a realization of the same dynamics in **one dimension**.
- A **holographic duality**: **near**AdS_2/**near**CFT_1 correspondence.

Sacdev, Ye '93, Kitaev '16; Maldacena, Stanford '16.

Broken Scale Invariance

- A 1D theory (quantum mechanics) in appropriate universality class: **the SYK-model**.

- A 2D theory in appropriate universality class: **Jackiw-Teitelboim gravity**.

- Either way: scale invariant IR limit is **trivial**.

  The quantum effective field theory describes the **breaking of scale invariance** by the **near IR** theory.

- Presently: **dimensionful order parameters** heat capacity C_T **and** capacitance C_φ break $\mathcal{N} = 2$ superconformal invariance.

Schwarzian Description of $\mathcal{N} = 2$ Superconformal Breaking

- The **Schwarzian** effective theory of broken scale invariance

$$I = -C \int_{\partial D} du \left[\frac{\partial_u^3 f}{\partial_u f} - \frac{3}{2} \left(\frac{\partial_u^2 f}{\partial_u f} \right) \right]$$

  The dimensionful coupling constant C is the **heat capacity**.

- The effective $1D$ theory of broken $\mathcal{N} = 2$ **superconformal invariance** adds

$$I = -C \int_{\partial D} du \; 2(\partial_\tau \sigma)^2$$

  The dimensionful coupling constant C is the **capacitance**.

- Upshot: the agreement $C_T = C_\varphi$ follows from **spontaneously broken** $\mathcal{N} = 2$ superconformal symmetry.

Fu, Gaiotto, Maldacena, Sachdev '16

Supersymmetric Index: not so Recent Developments

- Gravity = strongly coupled regime of the dual gauge theory.
- Foundation of reliable analysis: **protected states**.
- **Preserved** supersymmetry allows construction of the **supersymmetric index**:

$$I(\Delta_I, \omega_a) = \mathrm{Tr}[(-)^F e^{\Delta_I Q^I + \omega_i J^i}]$$

- Grading $(-)^F$ computes (bosons - fermions) $\Rightarrow$ certain **short representations** remain independent of coupling.
- Conventional wisdom: index $\mathcal{O}(1)$ (**confined** phace).

  Insensitive to black holes $\mathcal{O}(N^2)$ (**deconfined** phase).

Romelsberger '05; Kinney, Maldacena, Minwalla, Raju '05

Black Hole Entropy: Recent Claims

Partition function increases as $\mathcal{O}(N^2)$:

$$\ln Z = -\frac{N^2}{2}\frac{\Delta_1\Delta_2\Delta_3}{\omega_a\omega_b}$$

Insert $(-)^F \Leftrightarrow$ implement BPS condition by **complex constraint**

$$\Delta_1 + \Delta_2 + \Delta_3 - \omega_a - \omega_b = 2\pi i$$

$$\underset{\text{Legendre transform } \ln Z}{\Rightarrow} \quad S(Q^I, J^i) = \underbrace{\ln Z - \Delta_I Q^I - \omega_i J^i}_{\Delta_I,\omega_i \text{ at extremum } \textbf{subject to constraint}}$$

Result:

$$\mathrm{Re}\; S(Q^I, J^i) = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_1Q_3 - \frac{1}{2}N^2(J_a + J_b)}$$

$$\mathrm{Im}\; S(Q^I, J^i) = 0 \Leftrightarrow \textbf{constraint on conserved charges}$$

Index Computations: Strategy

- **Enumeration of free fields**: single fields (letters), composite fields (words), exponentiation (sentences?), singlet condition $\Rightarrow$ **unitary matrix model**

$$Z(\Delta_I, \omega_i) = \int dU \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} f(n\Delta_I, n\omega_i) \mathrm{Tr}\, U \mathrm{Tr}\, U^\dagger\right]$$

$$f(\Delta_I, \omega_i) = 1 - \frac{\prod_I (1 - e^{-\Delta_I})}{(1 - e^{-\omega_a})(1 - e^{-\omega_b})}$$

- **Supersymmetric localization** (*ab initio* or via Bethe vacua)
- ...

Upshot: consolidation using modern technology.

Contentious point: **asymptotic behavior** at large N.

Asymptotic Behavior of Matrix Model

- KMMR: single particle index $f < 1 \Rightarrow$ eigenvalues **repulsion dominates** $\Rightarrow$ **no condensation**.

- New result "Cardy limit" (simple but justification dubious):

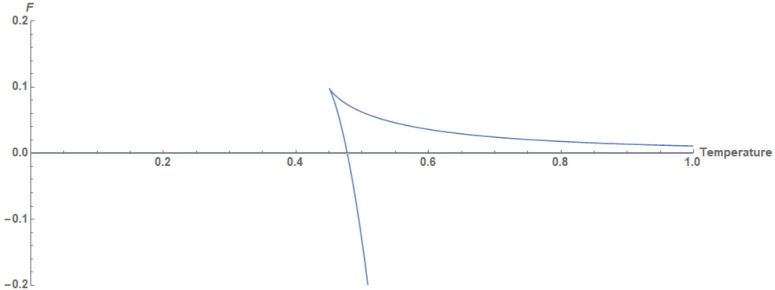
$$\sum_{n=1} \frac{1}{n}[1-f(\Delta,\omega)]\mathrm{Tr}U\mathrm{Tr}U^{\dagger} \underset{\omega_{a,b}\ll 1}{\longrightarrow} \underbrace{N^2}_{\mathrm{rank}\ SU(N)} \frac{1}{\omega_a\omega_b} \sum_{n=1,\pm} \frac{e^{\pm\Delta_1\pm\Delta_2\pm\Delta_3}}{n^3}$$

- Better new result: **modular invariance** in 4D SCFT Gadde '20
Also boils down to "maximal condensation": $\mathrm{Tr}U\mathrm{Tr}U^{\dagger} \to N^2$.

Key subtlety: study **complex** potentials.

Deconfinement?

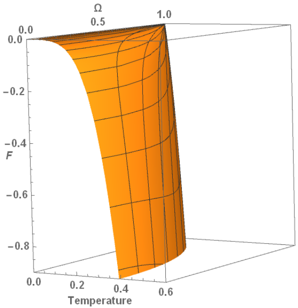
- The classical limit $Q_I, J_{a,b}, M \sim N^2 \gg 1$ is **deconfined**.
- Physics question: is the **low temperature phase** confined?
- AdS-Schwarzchild: **large BH branch** $(F < 0)$ does **not** reach $T = 0 \;\Rightarrow\;$ **confinement transition** to AdS-gas at $T < T_{\mathrm{HP}}$.



No Evidence of Phase Transition

- BPS surface has free energy $F \equiv 0$ (marginal bound state). and co-dimension 2: $T = 0$ and $\varphi = 0$.

- **No evidence of phase transition** ($F < 0$ throughout) when potentials are large $\varphi \geq 0 \Leftrightarrow \Omega \leq 1$.



BPS as a Limit

- The **partition function** (with real potentials)

$$Z = \mathrm{Tr}\,[e^{-\beta(E-E^*)+(\Phi_I-\Phi_I^*)Q_I+(\Omega_i-\Omega_i^*)J_i}] \underset{\mathrm{BPS}}{=} \mathrm{Tr}\,[e^{\Delta_I Q_I+\omega_i J_i}]$$

  BPS reference values are $\Phi_I^* = 1, \Omega_i^* = 1$.

- Low temperature limit ($\beta = \infty$) identifies

$$\begin{aligned} \mathrm{Re}\,\Delta_I &= \beta(\Phi_I - \Phi_I^*) = \partial_T \Phi_I \\ \mathrm{Re}\,\omega_i &= \beta(\Omega_i - \Omega_i^*) = \partial_T \Omega_i \end{aligned}$$

- Values of **thermal derivatives** ∂_T computed in spacetime/from microscopic free energy in fact **agree**.

Beyond Supersymmetry

- The **index**: insert $(-)^F$ **or complexify potentials**:

$$\Delta_1 + \Delta_2 + \Delta_3 - \omega_a - \omega_b = 2\pi i$$

- Minimal physical assumption: count "same" degrees of freedom also beyond the BPS limit.

- **Extrapolation of constraint** to the nearBPS regime:

$$\sum_I (\Phi_I - \Phi_I^*) - \sum_i (\Omega_i - \Omega_i^*) = \varphi + 2\pi i T$$

- Interpretation: the imaginary parts $\mathrm{Im}\ \Delta_I$, $\mathrm{Im}\ \omega_a$ **probe violation of the constraint**.

Supersymmetry Breaking is Protected

- Extremization of the entropy function with the generalized constraint is straightforward.
- It accounts for the parameters of **broken $\mathcal{N} = 2$ superconformal symmetry**.
- Example: the coefficient in the $\mathcal{N} = 2$ Schwarzian description

$$\frac{C_T}{T} = \frac{C_\varphi}{T} = Q^I \mathrm{Im}\, \Delta_I + J^a \mathrm{Im}\, \omega_a = \frac{8Q^3 + \frac{1}{4}N^4(J_1 + J_2)}{\frac{1}{4}N^4 + \frac{1}{2}N^2(6Q - J_1 - J_2) + 12Q^2}$$

Summary

- We developed aspects of AdS_5 black hole thermodynamics.

  Focus: the BPS limit and **near the BPS** limit.

- Highlight: **heat capacity** and **capacitance** agree.

  Interpretation: $\mathcal{N} = 2$ extension of **broken scale invariance**.

- Highlight: may **deform BPS constraint** between charges.

  Interpretation: **deform complex constraint on potentials**.