Microscopic Entropy of AdS Black Holes

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Microscopics of Black Hole Entropy

- The Bekenstein-Hawking area law for black hole entropy:
  \[ S = \frac{A}{4G_N} \]

- In favorable cases string theory offers a microscopic interpretation of the black hole: specific constituents, ...

- Statistical understanding \( S = \ln \Omega \) of the area law and more: higher dimension operators, quantum corrections, ...

- These developments are among the most prominent successes of string theory as a theory of quantum gravity.
AdS$_5$ Holography

- The best studied example of holography: String theory on AdS$_5 \times S^5$ is dual to N=4 SYM in $D = 4$.

- **Microscopic details** well understood (Quantum Field Theory!)

- The area law entropy of black holes in AdS$_5$ is a crude target: just the asymptotic density of states.

- Yet: only recently were quantitative agreements established in this context.

Hosseini, Hristov, and Zaffaroni 1705.05383
Cabo-Bizet, Cassani, Martelli, and Murthy 1810.11442
Choi, Kim, Kim, and Nahmgoong 1810.12067
Benini and Milan 1811.04017
Zaffaroni (review) 1902.07176.
This Talk

Overall focus: **Supersymmetric** AdS$_5$ black holes and their **nearBPS** relatives.

Outline:
- Black hole **thermodynamics**: “phenomenology”.
- Lessons from black holes in AdS$_3$.
- Relation to nAdS$_2$/CFT$_1$ correspondence.
- **Structure** of microscopic theory $\Rightarrow$ some puzzles.

Ongoing research (supported by DoE) with Sangmin Choi, Nizar Ezroura, Junho Hong, Siuyl Lee, Billy Liu, Jim Liu, Jun Nian, Shruti Paranjape, Yangwenxiao Zeng.
Quantum Numbers

- Geometry: $\text{AdS}_5 \times S^5$ has (superconformal extension of) $SO(2, 4) \times SO(6)$ symmetry.

- Fields in $SO(2, 4)$ representations: conformal weight $E$ and angular momenta $J_{a,b}$.

- Fields in $SO(6)$ representations: R-charges $Q_I$ ($I = 1, 2, 3$).

- So asymptotic data of (electric) black holes in $\text{AdS}_5$: Mass $M$, Angular momenta $J_{a,b}$, and 3 R-charges $Q_I$. 
Classical Black Hole Solutions

▶ General supergravity solution (Wu ’11).
  
  Independent mass $M$, angular momenta $J_{a,b}$, R-charges $Q_I$.

  Not widely known (and exceptionally complicated).

▶ General BPS (supersymmetric) solution: Gutowski+Reall ’05.

▶ BPS mass = “ground state energy” ($g = \ell_5^{-1}$):

$$M = \sum_{I} Q_I + g(J_a + J_b)$$

Novel features (not shared by asymptotically flat black holes):

▶ Only 2 SUSY’s preserved $= \frac{1}{16}$ of maximal.

▶ Quantum numbers $Q_I, J_a, J_b$ are related by a nonlinear constraint. Specifically, rotation is mandatory.
The Black Hole Entropy (BPS limit)

\[ S = 2\pi \sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3 - \frac{1}{2}N^2(J_a + J_b)} \]

- \( Q_I, J_{a,b} = \text{integral} \) charges. \( N = \text{rank} \) of dual gauge group.

- There are two scales in the problem: \( g = \ell_5^{-1} \) and \( G_5 \).

- They are \text{related as} \( \frac{\pi}{4G_5}\ell_5^3 = \frac{1}{2}N^2 \).

- \text{Classical charges} are \( \sim N^2 \) so the entropy is also \( \sim N^2 \).
The Constraint on Conserved Charges

BPS black holes all have charges satisfying:

\[ h \equiv \left( (Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3) - \frac{1}{2} N^2 (J_a + J_b) \right) \left( \frac{1}{2} N^2 + (Q_1 + Q_2 + Q_3) \right) \]

\[ - \frac{1}{2} N^2 J_a J_b + Q_1 Q_2 Q_3 = 0 \]

Corollary: **two** distinct deformations break supersymmetry

- Recall: \( T = 0 \) \( \iff \) extremality \( M = M_{\text{ext}} \) (lowest mass (given conserved charges))

- Standard SUSY breaking: **mass exceeds** \( M_{\text{ext}} \).
  Description: raise the temperature \( \Rightarrow T > 0 \).

- **Alternative**: violate constraint by adjusting conserved charges while preserving \( T = 0 \) (retain \( M = M_{\text{ext}} \)).
Constraint Follows from Supersymmetry

- (Inaccurate) lore: constraint required to avoid **naked closed timelike curves**.

- SUSY algebra + **unitarity** gives **BPS bound**:

\[
\{Q, Q^\dagger\} = M - M_{\text{BPS}} \geq 0
\]
\[
\sum_i Q_i + g(J_a + J_b)
\]

- Mass \(M\) of all black hole solutions satisfies identity with form

\[
M - M_{\text{BPS}} = (\ldots)^2 + (\ldots)^2 \geq 0
\]
\[
\equiv 0 \text{ for } T=0
\]

**BPS saturation** shows 2nd \((\ldots)^2 = 0\) \(\Rightarrow\) **constraint**.

- Constraint follows from **BPS** with **no other assumptions**.
Detour: BTZ Black Holes

Example: black holes in $\text{AdS}_3 \times S^3$ dual to $\text{CFT}_2$ with $(4,4)$ SUSY.

Analysis in $\text{AdS}_3$ spacetime and in $\text{CFT}_2$ are very similar.

**Four** quantum numbers: $\epsilon, p \, (\text{AdS}_3), \, j_R, j_L \, (S^3)$.

**Conformal weights** $h_{R,L} = \frac{1}{2}(\epsilon \pm p) + \frac{1}{4}k_{R,L}$ and **R-charges** $j_{R,L}$.

**Partition function:**

$$Z = \text{Tr} \, e^{2\pi i \tau (L_0 - \frac{1}{4}k_R) + 2\pi i z j_R - 2\pi i \bar{\tau} \tilde{L}_0 - \frac{1}{4}k_L + 2\pi i \bar{z} j_L}$$

$\text{SL}(2, \mathbb{R}) \times \text{SL}(2, \mathbb{R})$ invariant NS-**vacuum** $L_0, \tilde{L}_0 \rightarrow 0$ controlled by Casimir energy.
BTZ Black Holes: Statistical Description

Modular transform $\tau \to -\tau^{-1}$ maps vacuum to statistical regime:

$$\ln Z = \frac{\pi i k_R}{2\tau} (1 - 4z^2) - \frac{\pi i k_L}{2\bar{\tau}} (1 - 4\bar{z}^2)$$

Legendre transform gives the correct black hole entropy:

$$S = 2\pi \sqrt{k_R(E + P) - \frac{1}{4} J_R^2} + 2\pi \sqrt{k_L(E - P) - \frac{1}{4} J_L^2}$$

Extremality ($T = 0$): $\frac{1}{2}(E - P) = \frac{1}{4k_L} J_L^2$

BPS saturation (chiral primary):

$$\frac{1}{2}(E - P) + \frac{1}{4} k_L = \frac{1}{2} J_L \quad \Rightarrow \quad J_L = k_L$$

consistency

2nd condition
Perspectives on Constraint from AdS$_3$/CFT$_2$

> Supersymmetric states in CFT$_2$:
> **Chiral primaries** $\bar{h} = \frac{1}{2}j_L$ with $0 \leq j_L \leq 2k_L$ (unitarity).

> Supersymmetric black hole geometries:
> Exist only for $J_L = k_L$ (so **two conditions** on parameters).

> **Elliptic genus**: index entirely holomorphic $\Rightarrow$ co-dimension 2 in parameter space. Inserting $(-)^F$ “averages” over all $j_L$.

Physics lesson: the **constraint emerges** from supersymmetry of the **ensemble average**.
Another Perspective: SUSY Breaking Mechanisms

\[ S = 2\pi \sqrt{k_R(E + P) - \frac{1}{4}J_R^2} + 2\pi \sqrt{k_L(E - P) - \frac{1}{4}J_L^2} \]

SUSY breaking excitations

- **Conventional SUSY breaking** (temperature \( T \)):
  Activate excitations in the \( L \) sector.

- **Novel SUSY breaking**:
  \( L \) sector in **ground state**  \( \Rightarrow E > E_{BPS} = P + J_L - \frac{1}{2}k_L \).

  Additional excitations in the \( R \) sector.

Aside: 4D extremal Kerr breaks SUSY by the “novel” mechanism.
AdS$_5$ Black Hole: Heat Capacity

- Excite the black hole so mass above BPS bound

\[ M = M_{\text{BPS}} + \frac{1}{2} \left( \frac{C_T}{T} \right) T^2 \]

$C_T$ is the black hole **heat capacity** (proportional to $T$).

- Gravity computations give

\[
\frac{C_T}{T} = \frac{8Q^3 + \frac{1}{4}N^4(J_1 + J_2)}{\frac{1}{4}N^4 + \frac{1}{2}N^2(6Q - J_1 - J_2) + 12Q^2}
\]

- Interpretation: **number of degrees of freedom** in low energy excitations.

\[ \frac{C_T}{T} \] analogous to the **central charge** $c_L = 6k_L$. 

AdS$_5$ Black Hole: Capacitance

- BPS saturation implies the constraint so no SUSY if the constraint is violated.

- Then the black hole mass exceeds the BPS bound:

\[ M_{\text{ext}} = M_{\text{BPS}} + \frac{1}{2} \left( \frac{C\varphi}{T} \right) \left( \frac{\varphi}{2\pi} \right)^2. \]

- \( C\varphi \) is the capacitance of the black hole. (The potential \( \varphi \) is defined precisely later)

- Gravity computations give

\[ \frac{C\varphi}{T} = \frac{8Q^3 + \frac{1}{4}N^4(J_1 + J_2)}{\frac{1}{4}N^4 + \frac{1}{2}N^2(6Q^2 + J_1 + J_2) + 12Q^2}. \]

- **Note:** \( C\varphi = C_T \). Excitations violating the constraint “cost” the same as those violating the extremality bound!
nAdS$_2$/CFT$_1$ Correspondence

- All BPS black holes have $\text{AdS}_2$ near horizon geometry.

- AdS$_2$ does not allow excitations (with finite energy): they always deform the AdS$_2$ geometry.

- This strong IR dynamics in two dimensions has a universal description in effective quantum field theory.

- There is a realization of the same dynamics in one dimension.

- A holographic duality: near AdS$_2$/near CFT$_1$ correspondence.

Sacdev, Ye '93, Kitaev '16; Maldacena, Stanford '16.
Broken Scale Invariance

- A 1D theory (quantum mechanics) in appropriate universality class: the SYK-model.

- A 2D theory in appropriate universality class: Jackiw-Teitelboim gravity.

- Either way: scale invariant IR limit is trivial. The quantum effective field theory describes the breaking of scale invariance by the near IR theory.

- Presently: dimensionful order parameters heat capacity $C_T$ and capacitance $C_\phi$ break $\mathcal{N} = 2$ superconformal invariance.
Schwarzian Description of $\mathcal{N} = 2$ Superconformal Breaking

- The **Schwarzian** effective theory of broken scale invariance

$$I = -C \int_{\partial D} du \left[ \frac{\partial^3 f}{\partial u f} - \frac{3}{2} \left( \frac{\partial^2 f}{\partial u f} \right) \right]$$

The dimensionful coupling constant $C$ is the **heat capacity**.

- The effective 1D theory of broken $\mathcal{N} = 2$ superconformal invariance adds

$$I = -C \int_{\partial D} du \ 2(\partial_\tau \sigma)^2$$

The dimensionful coupling constant $C$ is the **capacitance**.

- Upshot: the agreement $C_T = C_\varphi$ follows from spontaneously broken $\mathcal{N} = 2$ superconformal symmetry.

Fu, Gaiotto, Maldacena, Sachdev '16
Supersymmetric Index: not so Recent Developments

- Gravity = strongly coupled regime of the dual gauge theory.

- Foundation of reliable analysis: **protected states**.

- **Preserved** supersymmetry allows construction of the **supersymmetric index**:

  \[ I(\Delta_I, \omega_a) = \text{Tr}[(-)^F e^{\Delta_I Q^I + \omega_i J^i}] \]

- Grading \((-)^F\) computes (bosons - fermions) \(\Rightarrow\) certain **short representations** remain independent of coupling.

- Conventional wisdom: index \(\mathcal{O}(1)\) (**confined** phase).

  **Insensitive to black holes** \(\mathcal{O}(N^2)\) (**deconfined** phase).

Romelsberger '05; Kinney, Maldacena, Minwalla, Raju '05
Black Hole Entropy: Recent Claims

Partition function increases as $O(N^2)$:

$$\ln Z = -\frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_a \omega_b}$$

Insert $(-)^F \Leftrightarrow$ implement BPS condition by complex constraint

$$\Delta_1 + \Delta_2 + \Delta_3 - \omega_a - \omega_b = 2\pi i$$

$\Rightarrow$ Legendre transform $\ln Z$

$$S(Q^I, J^i) = \ln Z - \Delta I Q^I - \omega_i J^i$$

$\Delta I, \omega_i$ at extremum subject to constraint

Result:

$$\text{Re } S(Q^I, J^i) = 2\pi \sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_1 Q_3 - \frac{1}{2} N^2 (J_a + J_b)}$$

$$\text{Im } S(Q^I, J^i) = 0 \Leftrightarrow \text{constraint on conserved charges}$$
Index Computations: Strategy

- **Enumeration of free fields**: single fields (letters), composite fields (words), exponentiation (sentences?), singlet condition

  ⇒  unitary matrix model

\[
Z(\Delta_I, \omega_i) = \int dU \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} f(n\Delta_I, n\omega_i) \text{Tr} U \text{Tr} U^\dagger \right]
\]

\[
f(\Delta_I, \omega_i) = 1 - \frac{\prod_I (1 - e^{-\Delta_I})}{(1 - e^{-\omega_a})(1 - e^{-\omega_b})}
\]

- **Supersymmetric localization**
  
  *(ab initio or via Bethe vacua)*

- ...  

Upshot: consolidation using modern technology.

Contentious point: *asymptotic behavior* at large \(N\).
Asymptotic Behavior of Matrix Model

- KMMR: single particle index $f < 1 \Rightarrow$ eigenvalues repulsion dominates $\Rightarrow$ no condensation.

- New result “Cardy limit” (simple but justification dubious):

$$
\sum_{n=1}^{\infty} \frac{1}{n} [1 - f(\Delta, \omega)] \text{Tr} U \text{Tr} U^\dagger \xrightarrow{\omega_{a,b} \ll 1} \frac{N^2}{\text{rank } SU(N)} \frac{1}{\omega_a \omega_b} \sum_{n=1,\pm} e^{\pm \Delta_1 \pm \Delta_2 \pm \Delta_3} \frac{1}{n^3}
$$

- Better new result: modular invariance in 4D SCFT Gadde ’20
  Also boils down to “maximal condensation”: $\text{Tr} U \text{Tr} U^\dagger \rightarrow N^2$.

Key subtlety: study complex potentials.
Deconfinement?

- The classical limit $Q_I, J_{a,b}, M \sim N^2 \gg 1$ is deconfined.

- Physics question: is the low temperature phase confined?

- AdS-Schwarzchild: large BH branch ($F < 0$) does not reach $T = 0 \Rightarrow$ confinement transition to AdS-gas at $T < T_{HP}$. 
No Evidence of Phase Transition

- BPS surface has free energy $F \equiv 0$ (marginal bound state) and co-dimension 2: $T = 0$ and $\varphi = 0$.

- **No evidence of phase transition** ($F < 0$ throughout) when potentials are large $\varphi \geq 0 \Leftrightarrow \Omega \leq 1$. 
BPS as a Limit

- The **partition function** (with real potentials)

\[
Z = \text{Tr} \left[ e^{-\beta(E-E^*)} + (\Phi_I - \Phi_I^*) Q_I + (\Omega_i - \Omega_i^*) J_i \right] = \text{BPS Tr} \left[ e^{\Delta I Q_I + \omega_i J_i} \right]
\]

BPS reference values are $\Phi_I^* = 1$, $\Omega_i^* = 1$.

- Low temperature limit ($\beta = \infty$) identifies

\[
\text{Re } \Delta_I = \beta(\Phi_I - \Phi_I^*) = \partial_T \Phi_I \\
\text{Re } \omega_i = \beta(\Omega_i - \Omega_i^*) = \partial_T \Omega_i
\]

- Values of **thermal derivatives** $\partial_T$ computed in spacetime/from microscopic free energy in fact **agree**.
Beyond Supersymmetry

- The index: insert \((-)^F\) or complexify potentials:

\[
\Delta_1 + \Delta_2 + \Delta_3 - \omega_a - \omega_b = 2\pi i
\]

- Minimal physical assumption: count “same” degrees of freedom also beyond the BPS limit.

- Extrapolation of constraint to the nearBPS regime:

\[
\sum_l (\Phi_l - \Phi_l^*) - \sum_i (\Omega_i - \Omega_i^*) = \varphi + 2\pi i T
\]

- Interpretation: the imaginary parts \(\text{Im} \, \Delta_l\), \(\text{Im} \, \omega_a\) probe violation of the constraint.
Supersymmetry Breaking is Protected

- Extremization of the entropy function with the generalized constraint is straightforward.

- It accounts for the parameters of **broken** $\mathcal{N} = 2$ superconformal symmetry.

- Example: the coefficient in the $\mathcal{N} = 2$ Schwarzian description

\[
\frac{C_T}{T} = \frac{C_\varphi}{T} = Q' \text{Im} \, \Delta_I + J^a \text{Im} \, \omega_a = \frac{8Q^3 + \frac{1}{4} N^4 (J_1 + J_2)}{\frac{1}{4} N^4 + \frac{1}{2} N^2 (6Q - J_1 - J_2) + 12Q^2}
\]
We developed aspects of AdS$_5$ black hole thermodynamics. Focus: the BPS limit and near the BPS limit.

Highlight: heat capacity and capacitance agree.

Interpretation: $\mathcal{N} = 2$ extension of broken scale invariance.

Highlight: may deform BPS constraint between charges.

Interpretation: deform complex constraint on potentials.