The No-Boundary Cosmological Measure

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A Quantum Universe

If the universe is a quantum mechanical system it has a quantum state. What is it?

A theory of the quantum state is the objective of Quantum Cosmology.
No State --- No Predictions

- The probability $p$ at time $t$ of an alternative represented by a projection $P(t)$ (e.g. a range of position) in a state $\ket{\Psi}$ is:

$$p = \left| \left| P(t) \ket{\Psi} \right| \right|^2$$

$$P(t) = e^{iHt/\hbar} P(0) e^{-iHt/\hbar}$$

- If we don’t have the operator $P$ and $H$ and the state $\ket{\Psi}$ there are no probabilities and no predictions.
Contemporary Final Theories Have Two Parts

An unfinished task of unification?
Third and First Person Probabilities

- The theory \((H, \Psi)\) predicts third person probabilities for which history of the universe occurs.

- First person probabilities for what we observe are third person probabilities conditioned on a description \(D\) of our observational situation --- including us. All we know is that there is at least one instance of \(D\). In a large universe \(D\) might be replicated.

\[ p(O|D^{\geq 1}) \]

- We test the theory by its first person predictions for what we observe.

\((H, \Psi)\) supply a probabilistic measure on cosmological histories.
3rd Person to 1st Person

Consider N Hubble volumes each with a probability $p_E(D)$ that our data evolved. All we know is that there is at least one Hubble volume with this data.

3rd person: $p(O)$, 1st person $p(O|D \geq 1)$

$$p(D \geq 1) = 1 - (1 - p_E(D))^N$$

$$p(O|D \geq 1) \propto [1 - (1 - p_E(D))^N]p(O)$$

The factor $1 - (1 - p_E(D))^N$ favors large universes over smaller ones, inflating universes over non-inflating ones, and eternally inflating universes over all others.
Are \((H, \Psi)\) enough to give 1st person probabilities for observations in all cases?

Or, are new laws of physics required in the form of a further measure.
Quantum systems behave classically when the 3rd person probabilities for histories with correlations in time governed by deterministic laws are high.
Classical Multiverses

- A simple, manageable, discoverable quantum state of the universe like the NBWF will not predict our unique classical history with all of its complexity.

- Rather it predicts an ensemble of possible classical universes with (3rd person) probabilities.
Basic Ingredients

- Decoherent Histories Quantum Mechanics.
- The no boundary quantum state
- A dynamical theory (H) with a single scalar field $\phi$ moving in a potential $V(\phi)$.
- Observers as physical systems within the universe with only a small probability $p_E$ to exist in any Hubble volume and, in a large universe, a probability to be replicated in many.
- A typicality assumption (xerographic distribution) relating us to the replicas as part of our model of our observational situation.
NBWF in Minisuperspace

Homogeneous, isotropic, and closed configurations of geometry and a scalar field.

\[ ds^2 = -dt^2 + a^2(t) d\Omega_3^2 \]

\[ \phi = \phi(t) \]

Dynamics: General Relativity, plus potential \( V(\phi) \)

No boundary wave function in the semiclassical approximation:

\[ \Psi = \Psi(b, \chi) \]

\[ \Psi_{NB}(b, \chi) \approx \exp[-I_{ext}(b, \chi)/\hbar] \]

The saddle point action is generally complex
3rd Person Predictions for Emergent Classical Histories

\[ \Psi(b, \chi) \approx \exp\left\{ \left[ -I_R(b, \chi) + iS(b, \chi) \right] / \hbar \right\} \]

WKB: When \( S \) varies rapidly compared to \( I_R \), an ensemble of classical histories is predicted that are the integral curves of \( S \).

\[ p_b = \nabla_b S \]
\[ p_\chi = \nabla_\chi S \]

3rd person probability of the history passing through \((b, \chi)\):

\[ p(\text{hist.}) \propto \exp\left[ -2I_R(b, \chi) / \hbar \right] \]

A one-parameter family labeled by \( \varphi_0 \).
Arrows of Time

• The growth of fluctuations defines an arrow of time, order into disorder.

• NBWF fluctuations vanish at the South Pole of the fuzzy instanton.

• Fluctuations therefore increase away from the bounce on both sides.

• Time’s arrow points in opposite directions on the opposite sides of the bounce.

• Events on one side will therefore have little effect on events on the other.

compare Caroll & Chen
Box Model Universe

Each box is Y, G, B with probabilities $p_Y$, $p_G$, $p_B$

Observers exist (E) with a probability $p_E$ in Y, G and 0 in B. We are equally likely to be any of the instances of E.

All boxes are statistically the same --- a symmetry

Probabilities of fine grained histories:

$$(p_Y)^{n_Y} (p_G)^{n_G} (1 - p_Y - p_G)^{N-n_Y-n_G} (p_E)^{n_E} (1 - p_E)^{N-n_E}.$$ 

Not well defined at infinite $N$
What is the 1st person probability we observe Y (WOY)?

We could try to sum the fine grained probabilities. But its easier to coarse grain over every box but ours.

\[ p(WOY) = \frac{p_Y}{p_Y + p_G} \]

The answer is well defined at large N with no further measure.
Why this Worked

- Because the model had the symmetry that is required to make predictions in an infinite or very large universe.
- Not a symmetry of each history but a statistical symmetry of the ensemble of histories of the kind naturally provided by quantum mechanics.
- Used coarse graining to directly calculate the 1st person probabilities for observation without calculating the large scale structure using a cutoff.
Quantum Nucleation of Bubbles of True Vacuum in a Classical False Vacuum
Multiverse of Bubbles

- (H) Einstein gravity coupled to a single scalar field.
- (Ψ) A quantum state.

- One false vacuum F and two true vacua A and B.
- Nucleation of true vacuum bubbles A or B by quantum tunneling dominate exit channels from F.
- Different slow roll regimes leading to different predictions for the CMB in A or B.
Not One Classical Spacetime but an Ensemble of Possible Ones

The state and its classical ensemble have deSitter symmetries which the individual histories do not.
Coarse Graining for Local Obs.

- Coarse grain of everything outside our bubble. Not by summing fine-grained probabilities over everything outside, but by summing amplitudes.

- Then there are only two histories. One in which our bubble nucleated somewhere, sometime, in true vacuum A and the other in true vacuum B.

- From the symmetries of deSitter these are the same as the probabilities that A or B nucleated in a particular place in spacetime.
Decoherence Enables Coarse Graining

Sum over amplitudes squared is the same as summing over probabilities.
The probabilities to nucleate bubbles of different kind in a false vacuum were calculated by Coleman and DeLuccia.

There is at least one copy of us in any bubble as long as \( p_E \) is not zero since the reheating surfaces are infinite.

The probabilities for which CMB we observe are:

\[
\frac{p(WOA)}{p(WOB)} = \frac{p_{CDL}(A)}{p_{CDL}(B)}
\]
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For this example no measure was necessary beyond that supplied by the NBWF.
Two Remarks
Deluded Observers

- Descartes’ evil demon
- Brain in a vat
- Matrix
- Boltzmann brains

To do physics we have to **assume** that we are not deluded. If you admit that you might be deluded, there is no reason to believe the data on which you base your theory. Admitting delusion is a self-undermining assumption.

If there are many more deluded brains than copies of us we have to assume that the xerographic distribution is such that we are **atypical**.

Assumptions of atypicality are possible because we have no observational evidence for typicality.
Does the Universe Have a Future?

- One dust grain in a superposition of two positions, deep in intergalactic space.
- Relative phases dissipate in of order $10^{-9}$ s from the $10^{11}$ CMB photons that scatter every second.

As the universe gets colder environmental decoherence gets less efficient,

$$t_{\text{decoh}} > .1 \text{sec} \quad \text{at} \quad t \approx 20t_H$$

Joos and Zeh ’85
An Extreme Example

\[ p_Y = p_G \quad p_E^Y = p_E^G / 1000 \]

In a typical instantiation there will be approximately equal numbers \( N_Y, N_G \) of yellow and green boxes but many more observers in green boxes than yellow ones.

\[ p(WOY) \equiv p(Y|E^\geq 1) = p_Y \quad E^\geq 1 \text{ means at least one instance.} \]

\[ p(WOG) \equiv p(G|E^\geq 1) = p_G \]

Provided:

\[ N_Y p_Y \gg 1 \quad N_G p_G \gg 1 \]
Probabilities from Counting in One History

\[ p_Y = p_G \quad p_Y^E = p_G^E / 1000 \]

If we are randomly selected from all observers then it's much more likely we will observe green because there are many more observers in green boxes than yellow.

There is no observational evidence that we are randomly selected and no physical mechanism for doing it.

The Selection Fallacy (H, Srednicki '07)
Main Points Again

• The universe has a quantum state that together with a theory of dynamics make a final theory.

• The state predicts not just one classical history but an ensemble of possible ones at different levels of coarse graining with 3rd person probabilities. It is a probabilistic measure.

• The state is naturally a source of statistical symmetries that enable prediction in very large or infinite universes.

• 1st person probabilities for or very coarse grained observations in our Hubble volume can be calculated in an appropriate coarse graining without first calculating a fine-grained picture of the large scale universe.
Is there a measure problem in inflationary cosmology?

YES

It's the problem of what is the quantum state of the universe.
H

- classical dynamics
- laboratory experiment eg CERN.

\[ H \]

\[ \psi \]

- classical spacetime
- early homo/iso + inflation
- fluctuations in ground state
- arrows of time
- CMB, large scale structure
- isolated systems
- topology of spacetime
- num. of large and small dims.
- num. of time dimensions
- coupling consts. eff. theories
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Scorecard for the No-Boundary Wave Function